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IRVINE

Flexible Management of Transportation Networks under Uncertainty

DISSERTATION

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for the degree of

DOCTOR OF PHILOSOPHY

in Civil Engineering

by

Joseph Ying Jun Chow

Dissertation Committee:
Professor Amelia C. Regan, Co-Chair
Professor R. Jayakrishnan, Co-Chair
Professor Jean-Daniel Saphores

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DEDICATION

To

my parents and my wife

for their unwavering love and support

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CURRICULUM VITAE

EDUCATION

2010	Ph.D. in Civil Engineering (Transportation Systems Engineering) University of California, Irvine, CA (GPA: 3.9)
2001	M.Eng. in Civil Engineering (Eng Mgmt/Systems Eng) Cornell University, Ithaca, NY (GPA: 3.7)
2000	B.S. in Civil Engineering (Minor Applied Math) Cornell University, Ithaca, NY (GPA: 3.4)

PEER-REVIEWED PUBLICATIONS

- Chow, J.Y.J. and A.C. Regan (2010). Real option pricing of network design investments. *Transportation Science*, in press.
- Chow, J.Y.J., C.H. Yang, and A.C. Regan (2010). State-of-the-Art of Freight Forecasting Modeling: Lessons Learned and the Road Ahead. *Transportation*, in press.
- Chow, J.Y.J., A.C. Regan and D.I. Arkhipov (2010). Fast converging global heuristic for continuous network design problem using radial basis functions. *Transportation Research Record: Journal of the Transportation Research Board*, in press.
- Chow, J.Y.J., G. Lee and I. Yang (2010). Estimating cumulative prospect theory parameters for HOV lane selection using genetic algorithm. *Transportation Research Record: Journal of the Transportation Research Board*, in press.

CONFERENCE PROCEEDINGS

- Chow, J.Y.J. and A.C. Regan (2010). Real option models for network design under uncertainty. *Proceedings from TRISTAN VII*, Tromsø, Norway, Jun 20-25 (forthcoming).
- Chow, J.Y.J., A.C. Regan and D.I. Arkhipov (2010). Fast converging global heuristic for continuous network design problem using radial basis functions. *Proceedings from the Transportation Research Board 89th Annual Meeting*, Washington DC, Jan 10-14.
- Chow, J.Y.J., G. Lee and I. Yang (2010). Estimating cumulative prospect theory parameters for HOV lane selection using genetic algorithm. *Proceedings from the Transportation Research Board 89th Annual Meeting*, Washington DC, Jan 10-14.

- Zhao, M., J.Y.J. Chow, and A.C. Regan (2009). Data for freight decision-making, *Proceedings from 3rd National Urban Freight Conference 2009*, Long Beach, CA, Oct 21-23.
- Chow, J.Y.J. and A.C. Regan (2009). Real option pricing of continuous network design investments. *Proceedings from the Transportation Research Board 88th Annual Meeting*, Washington DC, Jan 11-15.
- Yang, C.H., J.Y.J. Chow, and A.C. Regan (2009). State-of-the-Art of Freight Forecasting Modeling: Lessons Learned and the Road Ahead. *Proceedings from the Transportation Research Board 88th Annual Meeting*, Washington DC, Jan 11-15.

WORKING PAPERS

- Chow, J.Y.J. and A.C. Regan. Resource location and relocation models for statewide wildland fire planning. *INFOR*, revision under review.
- Chow, J.Y.J. and A.C. Regan. Multi-objective method for flexible robust network toll pricing with multi-regime network degradation. *European Journal of Operations Research*, under review.

SPONSORED RESEARCH REPORTS

- Chow, J.Y.J., S.V. Hernandez, A. Bhagat, and M.G. McNally (2010). *Sustainable Transit Feasibility Study for Mojave National Preserve*, #MOJA-00223, PI: Joseph Y.J. Chow, Sponsored by National Parks Conservation Association, 64pp.
- *Assessment and Development of Commodity Flow, Logistics, and Other Relevant Goods Movement Data Sources to Facilitate Statewide Freight Modeling* (2010), PI: Stephen G. Ritchie, Sponsored by California Department of Transportation.

CONFERENCE PRESENTATIONS

- "Multi-objective method for flexible robust network toll pricing with multi-regime network degradation", UCTC Student Conference, UC Irvine, Apr 1 – 2, 2010, forthcoming.
- "Fast Converging Global Heuristic for Continuous Network Design Problem Using Radial Basis Functions", CORS/INFORMS International Meeting, Toronto, CA, June 14-17, 2009.
- "Fire weather-based air tanker location and relocation models for statewide wildland fire planning", INFORMS 2009 Western Regional Conference, Tempe, AZ, April 24-25, 2009.
- "Real Options in Network Models under Uncertainty", Doctoral Student Research in Transportation Modeling, 2009 Transportation Research Board 88th Annual Meeting, Washington DC, Jan 11, 2009.

- "Real option pricing of continuous network design investments", UTC-PATH Conference: Tackling Congestion in an Era of Climate Change, Los Angeles CA, Nov 6 – 7, 2008.
- "Real options in network investment and operational risk hedging", UCTC Student Conference, UC Santa Barbara, Jan 31 – Feb 2, 2008.

TEACHING EXPERIENCE

University of California, Irvine – Irvine, CA

- Spring 2009: Teaching Assistant for Dr. Jin, CEE 11: Introduction to Probability, 147 undergrads enrolled, TA Evaluation 3.6/4.0

Cornell University – Ithaca, NY

- Spring 2001: Teaching Assistant for Dr. Meyburg, CEE 590: Project Management (grad), CEE 361: Intro to Transportation (undergrad)
- Fall 2000: Teaching Assistant for Dr. Davidson, CEE 598: Intro to Decision Analysis (grad)

WORK EXPERIENCE

09/2006 – Present University of California, Irvine – Irvine, CA
Graduate Student Researcher, Institute of Transportation Studies

- Statewide freight demand model scoping sponsored by Caltrans, forthcoming
- *Assessment and Development of Commodity Flow, Logistics, and Other Relevant Goods Movement Data Sources to Facilitate Statewide Freight Modeling*, PI: S.G. Ritchie and A.C. Regan, sponsored by Caltrans
- *Large Scale Real Options Models for Network Investment Planning and Operational Risk Hedging*, PI: A.C. Regan and J.Y.J. Chow, sponsored by University of California Transportation Center
- Alternative transit sustainability study for Mojave National Preserve, sponsored by the National Parks Conservation Association
- Market segmentation research for global ITS provider: on-site interviews with TMC managers on ITS features, regression and cluster analysis of demographics

07/2005 – 08/2006 Eng-Wong, Taub & Associates – New York, NY
Senior Engineer

09/2004 – 07/2005 Eng-Wong, Taub & Associates – New York, NY
Transportation Engineer

- Statewide demand model systems analysis: PennDOT Mobility Plan

- Traffic and parking design and impact studies: Stapleton Waterfront EIS, Willets Point EIS, Silvercup West Studios EIS, Yankee Stadium EIS, NASCAR Staten Island Raceway and Retail Center EIS, Jamaica Center Public Improvements Study, Bronx Terminal Market EIS
- Accident analysis: New York State Region 6, County Road 99 Corridor Study
- Pedestrian design: MetroNorth Transportation Center

10/2003 – 09/2004 Berger, Lehman & Associates – Rye, NY
Transportation Engineer

- Highway design: Interstate 84 Waterbury CT Reconstruction, Henry Hudson Pedestrian Parkway
- Accident analysis: Suffolk County Jericho Turnpike
- Signal design: Rye NY traffic signal survey and design

07/2001 – 10/2003 LOG-NET, Inc. – Little Silver, NJ
Project Coordinator

- Business/process analysis: US Customs 24-Hour Manifest, Commercial Invoice, Shipping Instructions, Bill of Lading, HTS Tariff, Load Planning, 304 EDI, 850/856 EDI
- Project management: LOG-NET 4.0-5.0 core product development

HONORS, FELLOWSHIPS, AND AFFILIATIONS

- Professional Engineer, NY
- Eisenhower Graduate Fellow, 2007 – 2010
- Eno Transportation Foundation Fellow, 2009
- Railway Club of Southern California Scholarship, 2008
- Nominated by Henry Samueli School of Engineering at University of California, Irvine, for Fletcher Jones Fellowship, 2008
- TRB, Networking Modeling Committee, Friend, 2007-Present
- TRB, Freight Transportation Planning and Logistics Committee, Friend, 2009-Present
- INFORMS, Member, 2007-Present
- TRB, Regional Transportation Systems Management & Operations Committee, Member, 2004 – 2008
- ITE Met Section, Co-Chair Student Outreach, 2003 – 2005, Chair NY Membership, 2005 – 2006
- Chi Epsilon, Member
- U.S. Citizen

ABSTRACT OF THE DISSERTATION

Flexible Management of Transportation Networks under Uncertainty

By

Joseph Ying Jun Chow

Doctor of Philosophy in Civil Engineering

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Professor Amelia C. Regan and Professor R. Jayakrishnan, Co-Chairs

Strategies, models, and algorithms facilitating such models are explored to provide transportation network managers and planners with more flexibility under uncertainty. Network design problems with non-stationary stochastic OD demand are formulated as real option investment problems and dynamic programming solution methodologies are used to obtain the value of flexibility to defer and re-design a network. The design premium is shown to reflect the opportunity cost of committing to a "preferred alternative" in transportation planning. Both network option and link option design problems are proposed with solution algorithms and tested on the classical Sioux Falls, SD network. Results indicate that allowing individual links to be deferred can have significant option value.

A resource relocation model using non-stationary stochastic variables as chance constraints is proposed. The model is applied to air tanker relocation for initial attack of wildfires in California, and results show that the flexibility to switch locations with non-

stationary stochastic variables providing 3-day or 7-day forecasts is more cost-effective than relocations without forecasting.

Due to the computational costs of these more complex network models, a faster converging heuristic based on radial basis functions is evaluated for continuous network design problems for the Anaheim, CA network with a 31-dimensional decision variable. The algorithm is further modified and then proven to converge for multi-objective problems. Compared to other popular multi-objective solution algorithms in the literature such as the genetic algorithm, the proposed multi-objective radial basis function algorithm is shown to be most effective.

The algorithm is applied to a flexible robust toll pricing problem, where toll pricing is proposed as a strategy to manage network robustness over multiple regimes of link capacity uncertainty. A link degradation simulation model is proposed that uses multivariate Bernoulli random variables to simulate correlated link failures. The solution to a multi-objective mean-variance toll pricing problem is obtained for the Sioux Falls network under low and high probability seasons, showing that the flexibility to adapt the Pareto set of toll solutions to changes in regime – e.g. hurricane seasons, security threat levels, etc – can increase value in terms of an epsilon indicator.

"Suppose that at each session, Congress shall first determine how much money can, for that year, be spared for improvements; then apportion that sum to the most important objects. So far all is easy; but how shall we determine which are the most important?" – Abraham Lincoln

CHAPTER 1 INTRODUCTION

Network design models have long been used to aid decision-makers in making investments on networks where the benefits cannot be clearly evaluated, such as in transportation planning. Decision-makers need to evaluate the success of their decisions using other "social benefit" measures, such as network link connection costs, or total vehicle-hours traveled. However, the change in today's environment is outgrowing the utility of the tools and strategies available to public decision-makers.

Lockwood (2005) expressed the primary problem in today's transportation agencies succinctly: "*The inherited culture of today's transportation agencies is dominated by facility development and preservation. Changes are required if state and* *local agencies are to have a significant impact on the characteristic 21st century mobility problems of congestion, unreliability, and insecurity*".

Mobility and accessibility certainly drove the goals of the Eisenhower administration in establishing the interstate highway system in 1956. However, the deeper problems suggested by Lockwood are the result of a maturing system and a more complex economic, political, and social environment. Early transportation agency leaders who focused almost solely on facility development and preservation could not have foreseen the need to deal with the complex issues of terrorist attacks or environmental justice. In the last two decades the need to transform the traditional transportation agency into an actively managing organization has received significant interest from both academic researchers and policy makers.

Logi (1999) and Mattingly (2000) showed that managing congestion can be much more effective when there is a more rational decision-making framework in place with better coordination between multiple decision-makers. Chung (2007) showed that mathematical models can be used effectively to manage non-recurrent congestion caused by accidents. Yang (2008) developed a concrete decision-making framework for evaluating truck management strategies such as lane restrictions, truck-only lanes, and virtual weigh stations. Apivatanagul (2008) illustrated the benefit of applying network design models to freight planning.

Whereas in the past an agency's primary objective was to allocate its budget to minimize travel costs in its network, today the same agency needs to manage a mature network to address other operational goals such as flexibility or robustness. These goals arise from more attention being placed on the transportation network because of issues such as fluctuating gas prices, natural disasters, and terrorist attacks.

1.1 MOTIVATION

The inadequacies of the state of practice are presented below using anecdotal examples. To avoid confusion, we begin by defining what we mean by *network management*. For the purposes of this research, network management is an act of control upon a system of connected nodes, arcs, and users with a goal of supporting the general welfare of the users over time. Whether this management requires long term planning, investment, or risk mitigation; short term pricing or resource allocation; or anything in between is subject to the decisions of the managing *agent*.

Although many kinds of networks exist, the focus in this body of work is on physical *transportation networks* traversed by people. These transportation networks are distinguished from other types such as electrical networks or utility networks by the different scale of capital costs; the longer time frame for arc traversal (and hence the focus on travel times between origin-destination (OD) pairs rather than on link capacities or bandwidth); and the ability of the travelers to make their own choices.

The examples shown below (and consequently the objectives of this research) represent a range of strategies available to a transportation network managing agent:

investment planning, resource allocation, and risk mitigation. Some of the terminology used is briefly discussed here but will be introduced in more detail in later chapters.

1.1.1 Investment Planning

Problems can be identified in the planning practice taken by many of the transportation agencies in the United States. Major investment projects at various local or regional agencies are generally undertaken with some level of federal funding. These investments are planned using Major Investment Studies (MIS), which provide information on costs and benefits for multiple alternatives. Decisions on which projects to undertake are often made at the local level. Investment projects are driven by long term (25 year) Regional Transportation Plans (RTP) for each metropolitan planning organization (MPO). Short term (3 year) federal funding needs are prioritized in transportation improvement programs (TIP). The day-to-day operational scheduling and programming of tasks is developed in the Unified Work Program for Transportation (UWP) on an annual basis. (FHWA, 2008)

The MPO's for three of the largest cities in the U.S. include the New York Metropolitan Transportation Council (NYMTC), Southern California Association of Governments (SCAG), and the Chicago Metropolitan Agency for Planning (CMAP). The RTP's of these MPO's provide forecasts of performance measures up to the projected time horizon, so there is some measurable visibility to the improvements from the plans' projects based on travel demand models.

However, projects are typically listed based on the cost of the preferred alternative instead of also providing conditional alternatives (SCAG RTP 2008, NYMTC RTP 2005, CMAP RTP 2008). As such, there are no conditional branches based on uncertainties in the environment, such as origin-destination (OD) demand or transit ridership, prices of materials, interest rate fluctuations, funding availability and size, or the impact of external policies on freight flows such as fuel taxes.

This problem of uncertainty in investment planning could be alleviated using a corporate finance technique called *real options*. It is essentially a dynamic programming method that evaluates a decision to choose between multiple operating modes given an asymmetric cost to switch between modes and stochastic value of the investment return over time.

Garvin and Cheah (2004) examine the benefit of a real options approach to the infrastructure investment industry, using a case study of the Dulles Greenway toll project in the 1990's as an example where the deferral option would have added more value to the decision-makers' investment.

In the case study, the planning agency initially estimated a demand of 20,000 vehicles per day for the first year of operation. Using traditional discounted cash flow approach, the static net present value (NPV) would have been -\$86.3M. However, real option analysis would include the value of deferral, resulting in an adjusted value of -\$86.3M + \$111.8M = \$25.5M. The static NPV approach should have suggested that the agency forego the project altogether. The real option approach would have informed the agency that there is potential value (\$25.5M), but much of it is from deferring the investment ($111.8M) and gathering more information on the uncertain elements such as the daily traffic flow.

It turned out the actual initial volumes were as low as 10,000 vehicles per day, forcing the agency to take more loans to cover their expenses. A real option approach at the beginning of the design or project approval period would have allowed the agency to time their investment better.

Furthermore, Garvin and Cheah's case study did not explicitly consider the value of alternative designs. If a network design approach was taken to estimate the value from users, then there would have been additional value from the flexibility to redesign the solution due to deferral. The investment opportunity could have exceeded $25.5M, where any difference would represent the value of NOT committing to their preferred alternative.

1.1.2 Resource Allocation

Resource allocation is a management tactic that can be applied at many different time frames. Long term planning examples include locating facilities to maximize the performance of a system, or even the proper allocation of funds to invest in new infrastructure. Short term operational examples include allocating green time to signalized intersections along certain congested corridors or positioning incident management assets to minimize the clearance time of a non-recurring incident.

Resource allocation for incident management in urban transportation networks can involve mitigating the impacts of accidents on freeways and arterials, and can also be applied to regional networks – for example, locating mobile resources to prevent wild fires.

Recent fires in Southern California dealt a devastating blow to the environment and the population in 2007. In San Diego County alone, the fires "claimed ten lives, destroyed more than 1,700 homes, burned more than 300 square miles and forced the evacuation of an estimated 500,000 people" (USA Today, Feb 19, 2008). A recent Los Angeles Times article (Boxall, December 31, 2008) claims that the cost of combating wildland fires in California now exceeds $1 billion.

This type of headline is becoming ever-more prominent, and the fire situation is likely to worsen in the future. In Rapp's (2004) study, nation-wide climate trends were modeled using annual data from 1895 to the present day and forecasted for the next 100 years. Results indicate that the southwestern United States will have wetter winters and warmer summers, leading to more woodland growth in the forests and more grass in the desert regions. These conditions will increase the risk of fire outbreaks.

Efficient fire and forestry management will only become more crucial over time. In California, this role is handled by the Department of Forestry and Fire Protection (CDF). The state is divided into 21 units under its control and 6 other counties in which fire protection is handled under individual private contracts. Fire protection plans developed by the agency involve stakeholder contributions, priorities, strategic areas for pre-fire planning and fuel treatment plans for local regions.

When the Santa Ana winds in Southern California caused a second major fire outbreak in Malibu in 2007, hundreds of firefighters and equipment from throughout the state were available to deploy to the region for a week in response to predictions about the winds (KNBC, 2007). On the other hand, fire authorities had insufficient resources at their disposal during the first outbreak several weeks prior, and needed to request last minute aid from neighboring regions. There is no systematic operational level model which uses daily fire weather data to optimally re-deploy resources.

If resource allocations can be actively managed using updated data and models that forecast near-term future conditions, decision-makers would be able to adapt their strategies and improve efficiencies. These improvements in efficiency can be measured and quantified as the value of pre-positioning resources based on potential future alternatives.

1.1.3 Risk Mitigation

Sheffi (2001) made a timely statement shortly after the events of 9/11 about how the western world is being ushered into a new era where large scale disruptions may occur. Using supply chain management as an example, he discusses the need for firms to juggle between two objectives: the traditional goal of minimizing cost with just-in-time inventory versus the need to maintain "*just-in-case*" inventory as well. Choosing only one leaves an organization more vulnerable to the effects of the other.

While the audience of the article was the corporate world, the message applies to the public sector as well: in the last ten years, the world has experienced other disruptions to the infrastructure from man-made acts of terror or conflict as well as natural disasters: Hurricane Katrina, the Indian Ocean Tsunami 2004, the 2005 London train bombings, the Northeast Blackout of 2003… the list goes on. In each case, the center of attention is the transportation or infrastructure network and how well it can cope with the additional stress on the system.

The complexity of managing transportation networks has increased significantly. Transportation planners and operational managers cannot focus solely on improving the efficiency of their networks for transporting people and goods; they must also heed other objectives such as minimizing environmental impacts or minimizing the sensitivity of the system's performance to uncertainties in supply and demand. Uncertainties in supply can arise from random incidents, such as an accident or a flood that closes off a roadway. Other uncertainties can arise from continual road deterioration, power failures affecting train operation, security threats, and more.

Certain network strategies are more applicable to managing risk with flexibility than others. For example, adding capacity to road networks is an irreversible investment that is difficult to adapt short term strategies to time-dependent uncertainties. On the other hand, some strategies such as adjusting ramp metering, signalization, or toll pricing can be used to adapt readily to new information on the state of a network.

By placing a toll on particular links or cordons in a network, it is possible for network managers to redirect traffic to reduce congestion throughout a network. In fact, there are a number of successful implementations of pricing schemes in cities such as Singapore, London, and Stockholm (Tsekeris and Voβ, 2008). However, there has been no research examining how toll pricing can be used to actively manage a network against uncertainties such as capacity degradation. Like the trade-off that supply chain managers need to make between minimizing cost and minimizing risk of loss, there exists a trade-off that can be made by network managers.

1.2 OBJECTIVE OF STUDY

The objective of this research is to apply innovative models and methodologies to improve upon flexible transportation network management strategies. As the anecdotal examples show, there is an inadequacy in the state of the practice in the way transportation networks can be managed under a setting of uncertainty. A number of different problem settings are explored with different ranges of flexibility strategies in the dissertation to provide decision-makers with a toolbox for flexible management of transportation networks. The bottom line is to incorporate adaptive decision-making using real option concepts to network-based managerial problems. A broad framework shown in FIGURE 1-1 is used to capture this toolbox.

The framework draws on de Neufvile's (2000) concept of a dynamic strategic planning for transportation networks and uses the definition of flexibility by Morlok and Chang (2004). These concepts are discussed in more detail in Chapter 2. Traditional network models and tools can be integrated with real options using the framework, which relates an objective to the uncertainties of concern, the setting of the network(s), the network controls available to the agent, and the type of option strategies represented in the strategies.

FIGURE 1-1. A framework for flexible transportation network management.

1.3 RESEARCH SUMMARY

This dissertation research examines several key aspects of flexible management of transportation networks under uncertainty and proposes new models and methods to support effective network design strategies. On the surface these proposed models may not appear to share much in common, but they support a framework that focuses on adaptive management of network resources based on gathering data and explicitly considering uncertainty.

In Chapter 2, the literature related to the role of network design models and real options in handling network-based strategies under uncertainty is explored. The review provides a background on the elements particular to the flexible transportation network management framework.

In Chapter 3, three network design-based option models are proposed. First, a network investment deferral option (NIDO) model is introduced to measure the option value of a network design investment with the option to defer. Second, a network option design problem (NODP) is proposed where the link investments are allocated to maximize the option value of a committed network design. Third, the network design is decomposed into individual, interacting link options as an ordered link investment deferral option set (OLIDOS). In this third model, the problem is to determine which links to invest immediately and which to defer, given a committed order to investing in them. Solution algorithms are proposed for the three models and applied to the classic example network of Sioux Falls, SD. A discussion of the solution algorithms and detailed analysis reveals the powerful capabilities available to an agent.

Chapter 3's option-based formulations and solutions represent long term planning in an urban freeway setting with non-stationary OD demand, using options to incorporate flexibility through deferral, design, and link interaction strategies.

Chapter 4 focuses on the value obtained from estimating time series data as a stochastic process for use in mobile server relocation problems. A resource relocation model is presented to show that incorporating predictive data in a resource relocation model can result in more cost effective relocation strategies because of the existence of hysteresis due to asymmetric relocation costs and high volatility in the demand. The model is applied to air tanker relocations in initial attacks against wild fires throughout California.

Chapter 4 features short-term resource allocation where the demand is derived from indices characterized as non-stationary mean-reverting processes. The setting is a regional transport network without congestion effects. The strategy is facility relocation representing the flexibility of changing the positions of resources based on the indices.

Chapter 5 focuses on the issue of computational efficiency with the need for globally optimal performance. A review of a relatively new global optimization algorithm based on radial basis functions is conducted. The algorithm is tested on a continuous network design problem to show that it would perform much more efficiently than the more traditional genetic algorithm approach. This algorithm is tested on both the Sioux Falls network under different congestion levels and the larger

Anaheim, CA network. Furthermore, the algorithm is modified and proven to handle multi-objective problems. The resulting MO-RBF algorithm is tested against results from the literature and shown to be very effective in comparison.

Chapter 6 examines the robust optimization problem for an urban road network as a tool for managing network robustness under different regimes or seasons of capacity uncertainty. A capacity degradation simulation model is proposed using multivariate Bernoulli random variables to represent link failure occurrences. The multi-objective algorithm proposed in Chapter 5 is used to solve a robust toll pricing problem with two objectives: maximizing expected social welfare versus minimizing variance of social welfare. The value of flexibility is quantified in terms of an epsilon indicator due to a change in the Pareto set in response to a change in scenario regime or season. The performance of the algorithm is tested on the Sioux Falls network to illustrate how an agent can incorporate flexibility into their robustness strategy.

Chapter 6 investigates the use of the toll pricing design strategy as a risk mitigation objective under stationary uncertainties in an urban freeway network, where the flexibility lies in being able to adapt a set of Pareto optimal robust toll prices to transitions in the uncertainties in link capacities from one mode, or regime, to another.

Chapter 7 ties all the pieces together and presents the contributions in terms of the framework. Future work is discussed.

1.4 RESEARCH CONTRIBUTIONS

The primary contributions of the research are summarized in the bullets below.

- By considering a network design with non-stationary stochastic OD demand as an option deferral problem, the solution incorporates the value to defer and to re-design the network as a function of the volatility in demand.
- The value of the network design premium is shown to represent the opportunity cost of giving up alternative designs when a transportation planner adopts a "preferred alternative".
- A network design problem is proposed to allocate funding to links to maximize its option value.
- A network design is shown to be a set of interacting link options, and when each is considered separately as an investment option the value can increase significantly. By committing the ordering of the link investments, the resulting model can be solved using a multi-option simulation algorithm.
- When time series data is characterized as a non-stationary stochastic process, it can provide additional information for network design problems in the form of chance constraints. This incorporation of chance constraints can be viewed as an explicit consideration of flexibility such as the hysteresis effect when positioning resources over time. This formulation of the resource relocation model is shown to perform better than a relocation model that does not make use of such forecasting.

- A radial basis function-based global optimization method is shown to be faster converging than the genetic algorithm for continuous network design problems for networks with up to 31 link investments.
- Modifications to the algorithm are made to allow it to solve multi-objective problems directly to obtain a Pareto optimal frontier. The algorithm is proven to converge and shown to be more efficient than existing multi-objective algorithms.
- A model of correlated link capacity degradation is developed that can be simulated with Monte Carlo methods.
- The value of flexibility in adapting a robust network design Pareto solution is quantified using an epsilon indicator. This value is used to illustrate toll pricing as a tool for managing network robustness with flexibility to adapt to changing regimes.

The implications of these findings are that the real option methodologies can bridge the gap between evolving public sector objectives and private sector tools and methods. They provide planners with a way of analytically considering uncertainty that can be visible to the public and to the decision-makers. A whole new area of research in network management using real option tools can evolve from this research to respond to the challenge posed by Mr. Lockwood shown in the beginning of the chapter.

Although the relocation modeling is set in a wildfire setting, the contribution has significant implications in many other related areas. For example, management of non-recurring incidents using real time traffic data can benefit from this development, although the relocation cost modeling would become more complex due to the congested links. The contribution can also benefit research in airline industries (flight delays from weather), freight delivery (multiple vehicle routing), and supply chain management (safety stock).

The modifications made to the earlier algorithm development pushes the radial basis function optimization method in new directions, particularly as a faster global solution method than more popular choices such as genetic algorithms or simulated annealing for network design models. In particular, the algorithm has been fine-tuned to handle multi-objective problems, which will find new applications in many areas.

The capacity degradation simulation and flexible robust optimization research offers a fresh take on an established network management strategy. Further applications in robust optimization, multi-objective problems, and other related network design problems can benefit significantly from this research.

"Doing dynamic strategic planning is comparable to playing chess: the planner thinks many moves ahead, but only commits to one move at a time" – Richard de Neufville

CHAPTER 2 NETWORK-BASED REAL OPTIONS REVIEW

As discussed, the goals of a transportation management agency have changed over the last few decades. This is true in a general sense as well; as de Neufville (2000) put it, the evolution of systems analysis has shifted from systems optimization in the 1970's, to decision analysis in the 1980's, to what he calls dynamic strategic planning in the current time. Dynamic strategic planning focuses on two primary aspects: the need to account for uncertainties, and the need to account for decision-makers.

This growing paradigm can be seen in transportation network management (TNM) as well. Many of the analytical tools used in TNM have evolved to reflect this trend, particularly network design models.

2.1 NETWORK DESIGN

Network design is a method of optimizing some objective in a network by making changes to the network structure. Besides the numerous types of networks that exist, it is frequently advantageous to think of many non-network problems using network structures (e.g. scheduling and supply chains).

Given a graph *G* of a set of nodes *N*, a set of links or arcs *A*, and a set of commodities *M*, subject to a set of constraints *S*, a network design problem (NDP) can be defined with the following generalized formulation shown in Magnanti and Wong (1984):

minimize $\phi(x, y)$ (2.1)

subject to

$$\sum_{j \in N} x_{ij}^m - \sum_{i \in N} x_{ji}^m = \begin{cases} R_m \; if \; i = O(m) \\ -R_m \; if \; i = D(m), \forall \, m \in M \\ 0 \; otherwise \end{cases} \quad (2.2)$$

$$x_{ij}^m \equiv \sum_{m \in M} x_{ij}^m \leq M_{ij} y_{ij}, \forall (i, j) \in A \quad (2.3)$$

$$(x, y) \in S \quad (2.4)$$

$$x_{ij}^m \geq 0, y_{ij} = 0 \; or \; 1, \forall \, (i, j) \in A, m \in M \quad (2.5)$$

Where *x* is the link flow for each commodity *m*, *y* is the design vector, R_m is the amount of the commodity located at each supply node.

The versatility of this model can be seen in the variety of related sub-problems derived from the formulation. For example, the objective can be defined as equation 2.6 (Ahuja et al, 1993):

$$\text{minimize } \phi(x,y) = \sum_{m\in M} c^m x^m + d^T y \tag{2.6}$$

Where *y* is a vector of links selected for design or construction, and *d* is the link design or construction cost vector. If R_m = 1 for all *m* and $x_{ij}^m \leq y_{ij}$ for all (i,j), this becomes an uncapacitated NDP.

2.1.1. Bi-Level Network Design

When congestion effects need to be accounted for, bi-level formulations are necessary, where the interaction of the design and link flow vectors with the objective leads to two separate problems. Although the design vector *y* is under the control of the decision-making agency, the behavior of the link flows *x* are typically not under their control. Using the formulation from the survey paper of Yang and Bell (1998):

$$\min_y \ \phi_1(x(y), y) \tag{2.7}$$

$$\text{subject to } S_1(x(y), y) \leq 0 \tag{2.8}$$

where $x(y)$ is implicitly defined by the following lower level problem:

$$\min_x \ \phi_2(x, y) \tag{2.9}$$

$$\text{subject to } S_2(x, y) \leq 0 \tag{2.10}$$

A thorough review of the different traffic equilibrium principles and solution methods can be found in Sheffi (1985), while a review of different types of bi-level urban network design problems and solution methods are available in Yang and Bell (1998) and recently in Apivatanagul (2008).

Among the bi-level NDP's, two distinct classes can be found: the discrete NDP (LeBlanc, 1975) where links or link components are counted as binary design variables, and the continuous NDP (CNDP) (Abdulaal and LeBlanc, 1979), where the capacity or other continuous link component can be improved incrementally. Variations of the bi-level NDP's such as signal timing (Ceylan and Bell, 2004), toll pricing (Yang and Bell, 1997), mass transport (Pagès et al, 2006), ramp metering, and reserve capacity can fall into one or the other class.

2.1.2. Network Design under Dynamic Strategic Planning Paradigm

With the shift to dynamic strategic planning, NDP's have evolved as well. Different objectives such as social and spatial equity (Yang and Zhang, 2002) consider a more heterogeneous set of users. Wei and Schonfeld (1994) solve a multi-period discrete network design staging problem using a neural network. Given a set of links, the problem is to determine the set of links as well as the time to invest in each such that the total travel costs over multiple periods are minimized, given that the demand is deterministic.

There is an abundant literature dealing with uncertainty in the many areas of network design, far more than can be described exhaustively here. Instead, some key research is mentioned among both the MILP's and the more complex bi-level problems. The examples mentioned below are mostly based on stationary stochastic variables, especially the bi-level problems. Unlike non-stationary stochastic processes where the probability distribution may evolve over time, a stationary stochastic variable has a fixed probability distribution. While stationary stochastic variables are simpler to solve, they cannot always reflect the dynamic strategies available to decision-makers over time.

Shu et al (2005) integrate the uncapacitated facility location model with an inventory problem under stochastic demand. Snyder et al (2007) extend this work to a generalized scenario approach that can account for discrete pseudo-non-stationary stochastic demand. In their model, stochastic demand can be modeled as a set of scenarios where each scenario represents a different year with different probability distribution of demand. The solution assigns locations for facilities in the initial time, and inventory policies are subject to change year to year. Snyder et al show that this stochastic location model with risk pooling (SLMRP) formulation can be solved with a Lagrangian-relaxation-based exact algorithm.

Although the demand would appear to be non-stationary with regards to the facility location decision variables, the retailer assignment and inventory management within each scenario has no foresight of near-term future demand because the formulation deterministically assigns parameters to each scenario with no dynamic updating. In other words, the demand is not truly an adapted process because the assignment at time t does not have better knowledge of where demand will be at time $t+1$. This model is sufficient if the cost of re-assigning retailers and inventory levels between years is negligible, but not if such reassignment incurs a significant cost. Chapter 4 explores this idea with a mobile server relocation problem using non-stationary stochastic demand.

Waller and Zilliaskopoulos (2001) and Chen and Yang (2004) approach the CNDP with stationary stochastic demand. The first assumes system optimal behavior to obtain an exact solution to the chance-constrained dynamic traffic assignment problem. The latter assumes user equilibrium behavior, but solves the CNDP with a heuristic genetic algorithm. Both objectives seek to minimize the expected value, or the first moment, of the objective function.

Karoonsoontawong and Waller (2007) consider higher moments with a robust formulation where the objective includes both the expected value and the variance of stationary stochastic variables. Robust optimization is covered in more detail in Chapter 6.

Sumalee et al (2006) model the NDP with an objective to maximize reliability of the total travel time. Reliability is defined in their paper as the probability that total network travel time falls below a threshold value. Naturally, their model could be extended to include a number of different reliability measures. Li et al (2007) explore optimal toll design with a reliability objective to examine a policy of using toll pricing to manage network reliability.

Whether the uncertainty is stationary or non-stationary, the solutions to the bi-level models above represent a static decision made at a single point in time. On the other hand, the SLMRP model serves as a good example of a model that can adapt to new information over time.

2.1.3. Flexibility

Morlok and Chang (2004) define flexibility as "the ability of a system to adapt to external changes, while maintaining satisfactory system performance". They also define external changes as "uncontrolled conditions that affect the system, including changes in level of demand or use, shifts in spatial traffic patterns, infrastructure loss and degradation, and changes in the price and availability of important resources such as fuel, etc."

Patil and Ukkusuri (2007) consider a flexible network design problem (FNDP) where the demand evolves as a non-stationary geometric Brownian motion. The idea is to allow the design to adapt to new conditions and to account for those possibilities by characterizing the non-stationary uncertainty with a stochastic process. However, they formulate the problem as a staging design problem with an adapted stochastic process (i.e. at any point in time, only the past is revealed).

Damnjanovic et al (2008) consider network flexibility from a different angle. They model the network design problem as a stationary stochastic program with recourse for the purpose of obtaining the value of recourse for a network compared to the same problem without recourse. While they do not use non-stationary uncertainty, they make use of the broad concept of real options in terms of measuring the value of flexibility for the purpose of decision-making under uncertainty. Among their conclusions is the idea to extend their work to multi-stage situations in which the next stage recourse would be nested in the previous stage structure.

The existing literature suggests that flexibility can be used to address network design models under uncertainty. It also reveals the complexity of such problems, especially the urban problems with a bi-level structure. However, in the world of corporate finance the concept of real options has grown significantly in the last few years as a tool for extending flexibility to projects under uncertainty. The solution methodologies developed in this field can be applied to network design models to establish a dynamic TNM framework where flexibility of strategies can be quantified and optimized.

2.2 REAL OPTIONS

Before defining real options, it is important to understand financial options. This is because the mechanics of real options are based on financial options, although the meaning of each type of option is very different from one other.

2.2.1 Financial Options

In finance, a call (put) option is a contract that one can purchase to give the owner the right to buy (sell) a stock at a specific target, or strike, price. There are many different kinds of options, but two kinds can be distinguished by the manner with which they can be exercised: a European option allows an owner to exercise the option only on the expiration date at some time *T*, whereas an American style option allows the owner to exercise the option at any time up to *T*. The following example is obtained from Hull (2006) and expanded upon.

Suppose an investor is interested in purchasing shares of Intel stock. At the end of May, its price is at $20.83. If there is no uncertainty at all and the stock price follows a deterministic path, then the option to purchase the stock at $22.50 by October should be determined by the deterministic growth rate. Assuming compound interest, the value of the stock at time *t* should be:

$$S(t) = S_0 e^{\rho t} \tag{2.11}$$

Where S_0 would be the initial price and ρ would be the interest or discount rate. Assuming it is a monthly interest rate for this example, then a ρ = 1.54% or lower would indicate a stock price that would be less than or equal to $22.50 by October. A contract to purchase the stock at $22.50 until October under the low interest rate would be worthless because the investor can just buy the stock for cheaper before that expiration date.

On the other hand, a higher ρ that would lead to a deterministic value of \$23.00 by October should be worth \$0.50 because the investor could then exercise the contract in October to buy the stock at \$22.50 and sell it at \$23.00 for a \$0.50 profit. This is assuming there is no arbitrage in the market, i.e. no such thing as a free lunch, because if the contract was cheaper it would be bought up and sold at \$0.50 for a profit.

The value of options as a tool for hedging risk comes when uncertainty is introduced. FIGURE 2-1 shows two scenarios. In the *Stock* scenario, the investor chooses to purchase 100 shares of Intel stock, and their profit can range from -\$2083 (the amount that they spent to purchase 100 shares at \$20.83) or up to \$1917 if the stock price hits \$40. Depending on the volatility of the stock, the downside risk can be tremendous.

On the other hand, the same investor can purchase a contract to buy 100 shares of Intel stock that expire in October at the strike price of \$22.50. Let's assume that the price of this option is \$1.15 per share, or \$115 for the 100 share contract. Then they are risking at most \$115 (because if the price dips below a profitable value, the investor can choose not to exercise the option), but can stand to gain up to \$1635 given an equivalent comparison of a \$40 stock price. As the example illustrates, options are a form of insurance to manage risk under a highly volatile environment. As volatility increases, the value of insurance should increase as well, leading to a relationship between the underlying uncertainty defining the stock price and its option value.

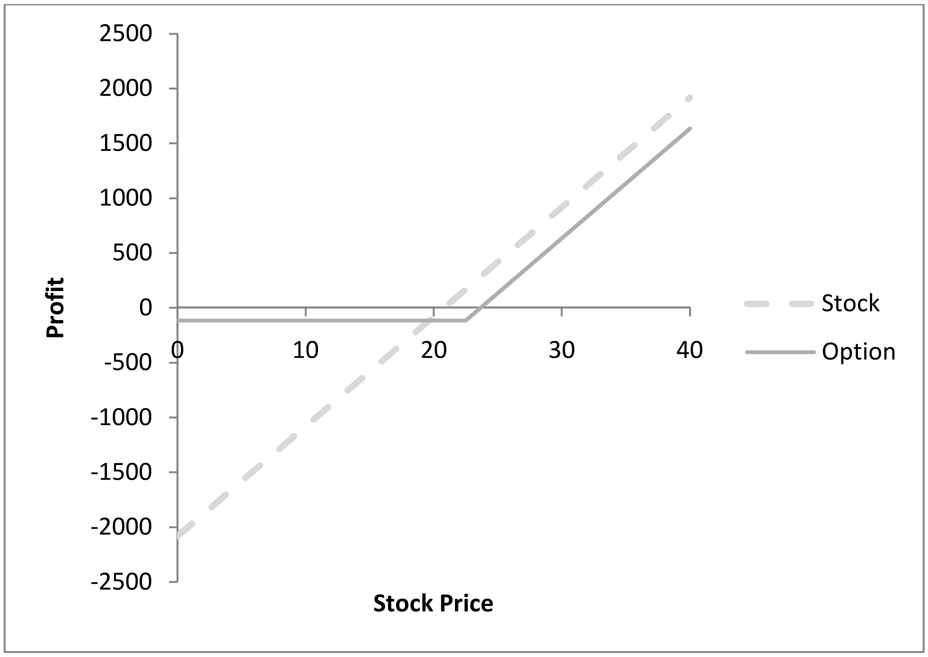

FIGURE 2-1 Comparison of stock purchase versus option exercise under uncertainty.

2.2.2 Stochastic Processes

To quantify this relationship between the option value and a stock price under uncertainty, the uncertainty needs to be defined as a measurable function over time. For the remainder of this section, stochastic processes are discussed from a continuous time perspective for consideration of closed form methods in Section 2.2.5.

As per Dixit and Pindyck (1994), a stochastic process is defined as "a variable that evolves over time in a way that is at least in part random". A non-stationary stochastic process is one where the distribution may change over time. An important concept is whether a process is *adapted*. Karatzas and Shreve (1998) define it as follows.

Definition 2.1. The stochastic process X is *adapted* to the filtration $\{\mathcal{F}_t\}$ if, for each $t \geq 0$, X_t is an $\mathcal{F}_t$-measurable random variable.

This means that an adapted stochastic process has a time dependent probability distribution that is based only on current and prior information at time *t*, and nothing afterwards. The definition is needed because it implies decision-makers that depend on adapted stochastic processes do not know what will happen in the future but have up-to-date information.

A basic building block for adapted processes is a Wiener process, also commonly called a Brownian motion. Dixit and Pindyck highlight three important properties that define a Wiener process:

1. A Wiener process is a Markov process, i.e. future probability distributions depend only on the current value;
2. A Wiener process has independent increments, i.e. probability distributions for the change in the process for two non-overlapping time periods are assumed independent of each other;
3. An increment of the Wiener process over a finite time interval is normally distributed with a variance that increases linearly with the interval.

Given that definition, a generalized Brownian motion or Ito process *x* can be defined as follows.

$$dx = a(x,t)dt + b(x,t)dW \qquad (2.12)$$

Where *dW* is an increment in a Wiener process and *a(x,t)* and *b(x,t)* are non-random functions. If *a* and *b* are constants, the resulting Brownian motion has an expected value of $E[x|a = \alpha, b = \sigma] = \alpha x_0 t$, a variance $V[x|a = \alpha, b = \sigma] = \sigma^2 t$, and it follows a normal distribution. More complex forms of *a* and *b* can be solved by using Ito's Lemma, a form of Taylor series expansion, to transform the stochastic process into a differential equation.

Two common forms of the Ito process are the geometric Brownian motion with drift and the Ornstein-Uhlenbeck mean-reverting process. The geometric Brownian motion with drift is defined by Dixit and Pindyck as:

$$dx = \alpha x dt + \sigma x dW \qquad (2.13)$$

Where the expected value of *x(t)* is $E[x(t)] = x_0 e^{\alpha t}$ and the variance is $V[x(t)] = x_0^2 e^{2\alpha t}(e^{\sigma^2 t} - 1)$. The distribution of the increment is lognormal, and it is commonly used for many variables that have a compound or exponential growth rate. FIGURE 2-2 illustrates one sample path of realization for a variable that follows a geometric Brownian motion and shows how the parameters of the process can be used to define an expected future trajectory from a current time along with a confidence interval.

The model introduced in Chapter 3 assumes the demand for each OD pair follows geometric Brownian motion.

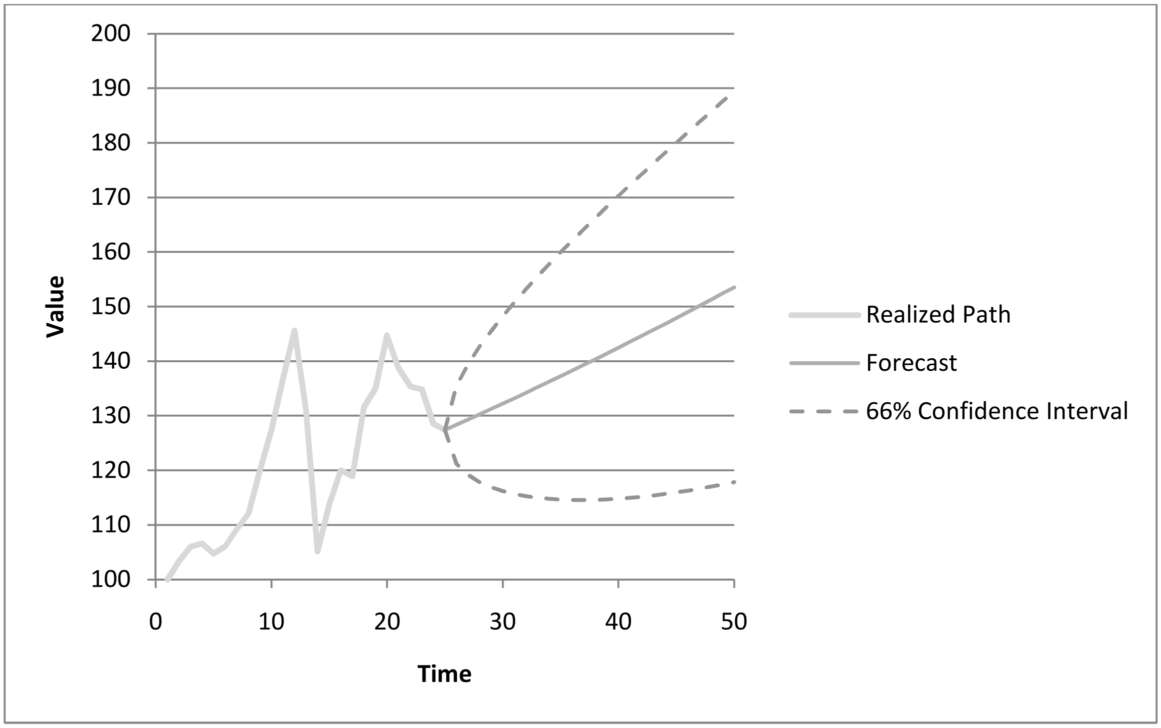

FIGURE 2-2. Example of geometric Brownian motion.

The Ornstein-Uhlenbeck (O-U) process, in its simplest form without drift can be defined as follows:

$$dx = \eta(\mu - x)dt + \sigma dW \qquad (2.14)$$

Where η is a rate of reversion towards the mean value μ. The fire weather indices in Chapter 4 are characterized as simple O-U processes. Many more types of processes exist, including Poisson jump processes for modeling discrete events with a mean arrival rate.

2.2.3 Types of Real Options

Recall the example from FIGURE 2-1. In the example, a financial option can hedge risk by mitigating downside loss. In strategic management, a decision-maker can assess the value of their decisions using the method of discounted cash flow to consider all their alternatives with equivalent units of present time value, or net present value (NPV). However, the NPV is the equivalent of immediately committing to the stock in the example in FIGURE 2-1, where the consequence of a strategic decision would be subject to the outcomes of uncertainty in the future.

In real life strategic planning, a decision-maker does not make static decisions and follow through with them regardless of intermediate outcomes. Instead, they incorporate flexibility into their planning by using current information to adjust their plans over time. However, the NPV method of evaluation does not reflect this behavior, just like a stock purchase would not reflect the manner with which an investor can maintain an option to purchase a stock to minimize risk of loss. A real option doesn't so much as hedge the risk in the same fashion as a financial option; instead, it is a construct used to model flexible decision-making behavior so that it can quantify the value of flexibility under uncertainty.

Real options cannot be treated in the same manner as financial options because of several differences. First of all, they cannot be traded as securities like financial options. Much of the value of financial options is due to the ability to trade them as much as it is to exercise them. Whereas a stock purchase is fairly liquid in the sense that it can be traded at any time, the other key difference is that many strategic decisions face highly asymmetric reversal costs. The condition of irreversibility in strategic management is where real options gain most of their value as a tool for quantifying flexibility for optimal dynamic decision-making over time under uncertainty.

Another major difference is that financial options are typically derived directly from a single stock's price; however, the value of an asset in real options may be a function of multiple *d*-dimensional variables, making it much more complex to resolve. For example, in the transportation examples the social welfare may be the objective value to maximize, but it is a function of equilibrium conditions due to stochastic OD demand or capacity.

Trigeorgis (1996) presents a framework for using real options in strategic management by expanding on the NPV:

$$Expanded\ NPV = NPV + Option\ premium \tag{2.15}$$

The NPV is expanded upon by incorporating the *option premium*, which represents the value due to options that a static NPV analysis would not be able to capture. Benaroch and Kauffman (2000) present a clear comparison of this valuation versus traditional discounted cash flow method as well as basic decision tree analysis in FIGURE 2-3.

They argue that real options offer two incentives for strategic management: 1) real options can model payoff contingencies using probability distribution functions so that the presence of an option can translate to expectations of shifts in the distribution; and 2) they can replace the actual probabilities of payoffs by risk-neutral probabilities to facilitate discounting by a risk-free rate instead of the risk-adjusted rate. For the purposes of this research, all the decision-makers are assumed to be risk-neutral to focus on the decision-making under uncertainty instead.

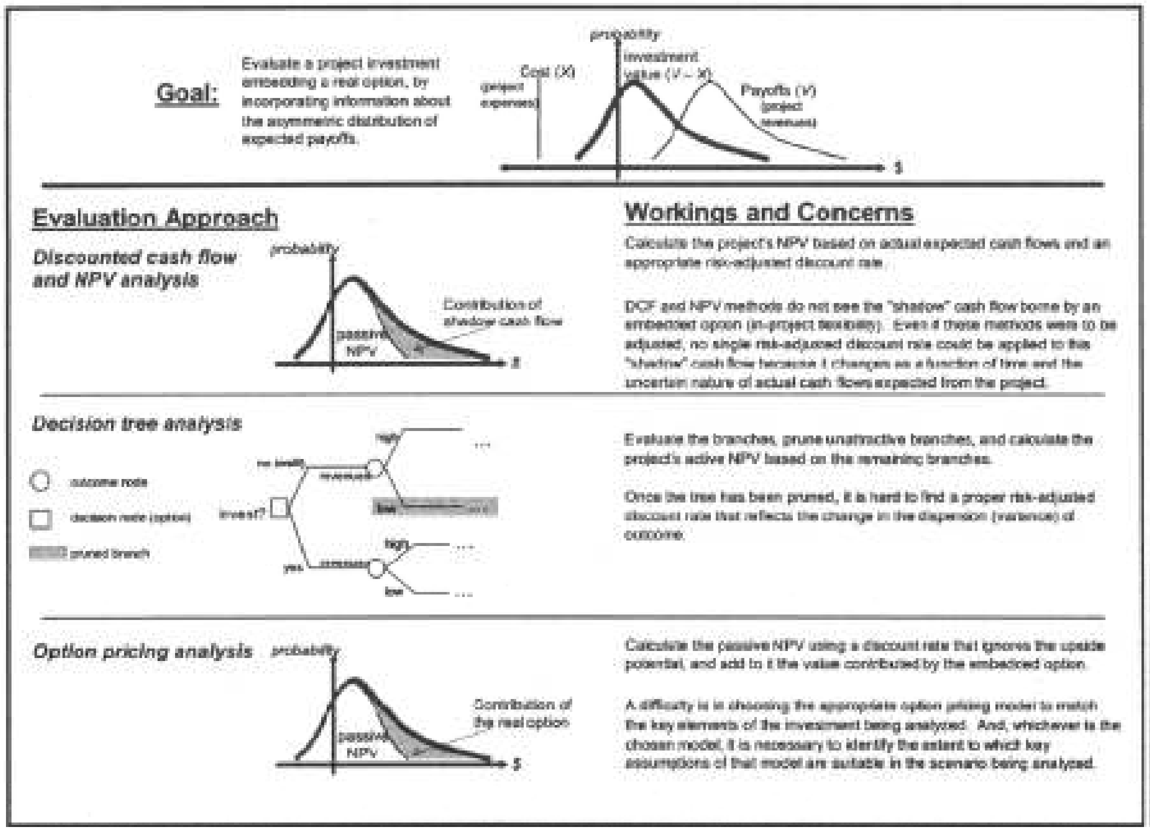

FIGURE 2-3. Comparison of different strategic evaluation methods. (Benaroch and Kauffman, 2000)

Luehrman (2001) makes a very intuitive argument for how real options can enhance strategic management from basic discounted cash flow analysis. He illustrates how the option space region expands from only zones A and C in FIGURE 2-4 for a static NPV analysis to the six regions using real options analysis (ROA) instead. Traditional TNM strategies confined to "now" or "never" can be expanded to include "maybe now", "probably later", "maybe later", and "probably never", depending on volatility and NPV.

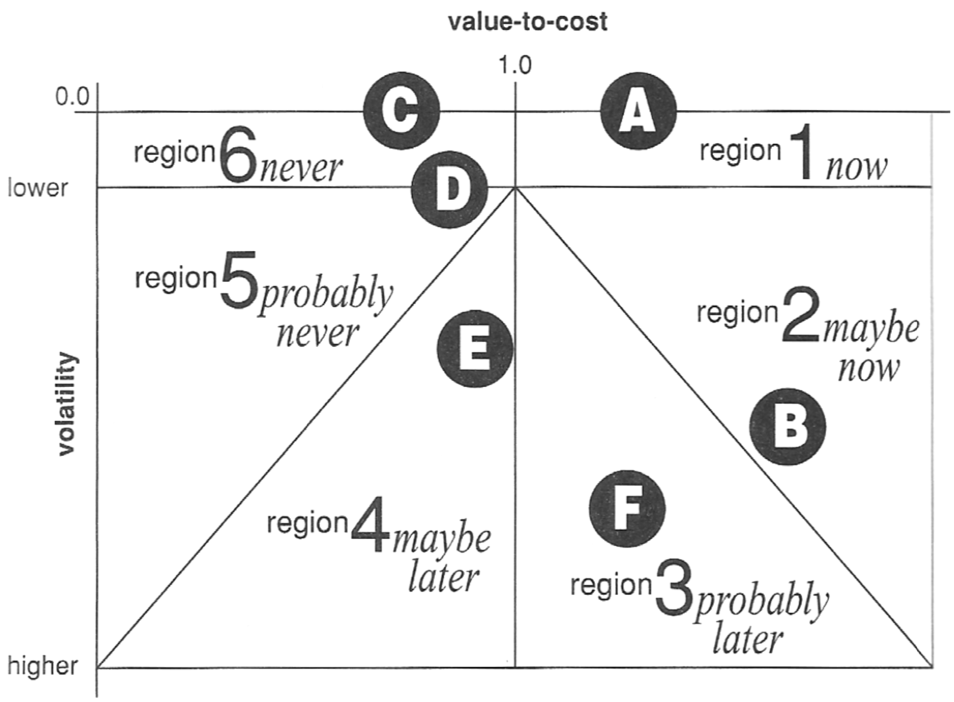

FIGURE 2-4. Expansion of NPV to ROA. (Luehrman, 2001)

There are many different kinds of real options that can fit into the *option premium* in the framework provided by Trigeorgis. For example, Benaroch (2002) lists the many types of real options that can apply to information technology (IT): deferral, staging, exploration, altering scale, abandonment, switching use, outsourcing, leasing, compound options, and strategic growth.

In addition to those options, Harmantzis et al (2006) introduces options specific to the telecommunications industry such as the option to discover or to franchise.

Kauffman and Kumar (2008) introduce the option to sponsor network technology investments. Damnjanovic et al (2008) identify the recourse option in a simplified two-stage stochastic setting.

Cucchiella and Gastaldi (2006) explore different real option types for managing risks in supply chains. Many of the risks that they categorize also arise in some variant form in TNM for public agents. For example, available capacity is an uncertainty in transportation planning, while information delays have significant weight in real-time traffic operations and incident management.

2.2.4 Application of Option Analysis to Networks and Transportation

Besides the different types of options that have been derived to model different management strategies for dealing with uncertainty, ROA has been applied to a number of network-related and transportation problems.

Kogut and Kulatilaka (1994) explore the value of switching between different suppliers in an international supply chain network when currency exchange rates fluctuate as a mean-reverting process. Although they ignore transport costs, their research introduces the concept of a "hysteresis band" representing a region of inertia between two modes of operation. In other words, when there is uncertainty and high switching costs, it is not always optimal to switch operation just when the threshold of a static NPV shows that the threshold has been crossed. This is because there is value in maintaining a current position in case conditions revert back in the near future. This band increases when there is greater uncertainty and higher switching costs. Chapter 4 investigates this notion of maintaining position in facility relocation problems for wildland fire resource planning.

Other relatively early adopters of ROA for network applications are Huchzermeier and Cohen (1996). They investigate the switching option for a supply chain network design by incorporating the design problem as a sublevel of the switching option valuation. Their model is shown in FIGURE 2-5. One of the conclusions that the authors make is that "supply chain network options differ from project options, because they exploit synergies derived from global coordination of multiple investments, i.e. network design decisions..." They show that there is value from the flexibility to redesign a supply network in the future. It is relevant to TNM because bi-level transportation network design problems can also be modeled using a similar fashion to increase value under uncertainty. This value from redesigning a network is explored further in Chapter 3.

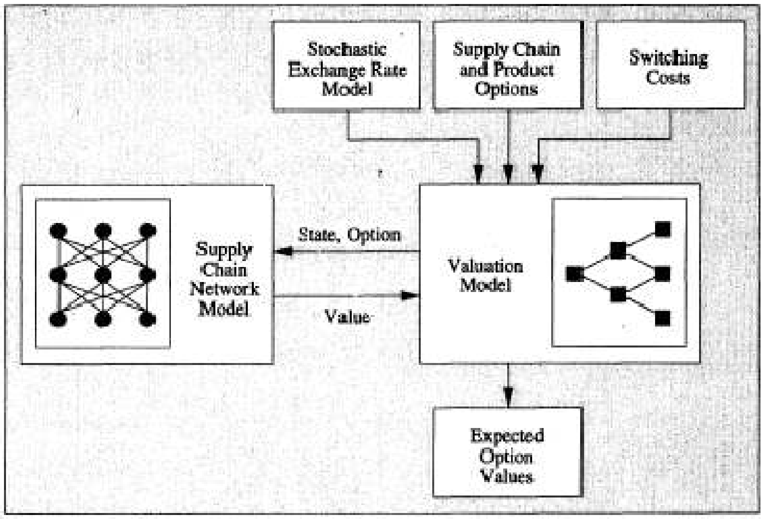

FIGURE 2-5. A deferral option model incorporating supply chain design. (Huchzermeier and Cohen, 1996)

Other ROA studies in networks include Keppo's (2002) option pricing for link bandwidths in a telecommunications network; Snyder's (2006) review of ROA as a tool for facility location under uncertainty; Cucchiella and Gastaldi (2006) on using ROA to manage supply chain risks; and Kauffman and Kumar (2008) on identifying network effects and embedded options in network technology investments. Snyder commented that efforts to incorporate real options have generally been to compute the option value only, not to incorporate the optimization of the option value as a function of design variables. This is achieved in Chapter 3 for the network design problem by maximizing the option value.

In addition to Dulles Greenway project example given by Garvin and Cheah (2004), other researchers have applied ROA to the transportation industry. Zhao et al (2004), Pichayapan et al (2003), and Vergara-Alert (2007) use real option approaches to highway investment under stochastic demand. Zhao et al's solution approach is based on Least Squares Monte Carlo simulation, Pichayapan et al's approach is based on a binomial method, and Vergara-Alert offers a closed form approach. The underlying methodology compares the benefit of a highway segment investment to the cost of construction and compares it to traditional NPV analysis. In a network setting, however, there are interdependencies in performance due to the interrelated stochastic flows that cannot be ignored.

Saphores and Boarnet (2006) investigate the application of real options in congestion relief investments from an economic and urban planning standpoint. They show that under complete certainty, investment decisions based on utility maximization are approximately equivalent to the policies derived from standard benefit-cost analysis.

Tsai et al (2008) construct an option contract for trading truckload contracts in the freight trucking industry. They use monthly truckload contract pricing data and apply statistical methods to estimate daily fluctuations in price for characterizing the stochastic prices as O-U processes.

While there are applications of real options in transportation, there hasn't been much effort in using the solution methodologies to incorporate flexibility in network design strategies that explicitly account for network effects.

2.2.5 Solution Methods

The discussion has so far been on the benefits of real option analysis, but not specifically on how they can be solved. Given an expiration time *T* and a project value based on one or more non-stationary stochastic processes, the problem of deciding the optimal time to exercise an option can be interpreted as an optimal stopping problem, according to Dixit and Pindyck (1994). The optimal stopping problem can be solved via dynamic programming when the interest rate ρ is specified exogenously. This is typically the case in public infrastructure where the benefit of a project is measured in terms of social welfare, which does not have equivalent market portfolios.

2.2.5.1 Discrete Time Optimal Stopping Problem

An optimal stopping problem is one class of dynamic programming control problems which involves optimizing the binary decision to continue a process for another incremental time period or to stop it at the current time period. The problem can involve an infinite time horizon in which case the steady state decision is evaluated; or a finite time horizon which is generally the case for real option problems. The problem is decomposed into a backwards dynamic program where the objective function in each time state is defined as follow.

$$\Phi_t(x_t) = \max_{u_t}\{\pi_t(x_t, u_t) + e^{-\rho\Delta t} E_t[\Phi_{t+\Delta t}(x_{t+\Delta t})]\} \qquad (2.16)$$

Where u_t is the decision to continue or stop the process at time t, π_t is the profit flow in the current time state as a function of u_t and state variable x_t, ρ is the discount rate for continuous compounding, E_t is the expectation at time t, and Φ_t is the value of the control problem. Eq. (2.16) is commonly called the Bellman equation or the fundamental equation of optimality (Dixit and Pindyck, 1994).

An example of a real option defined as a stopping problem is the deferral option. In this example, the process is deferral and in each time state the problem is whether to continue deferring or to stop deferring and to invest immediately with a value of Φ_0.

The problem can be solved using differential equations if the state variable is an Ito process. However, in many real option problems (and particularly for transportation network management problems) the state variable is not the Ito process, but a function of other variables that evolve as Ito processes. Vergara-Alert (2007) demonstrates the limits of closed form solutions for a transportation investment problem by solving a very small and simple problem which does not consider network effects. Clearly, numerical methods are necessary to solve realistic real option dynamic programming problems.

2.2.5.2 Numerical Methods

Trigeorgis (1996) describes several common numerical methods developed to handle real option analysis. The methods can generally be categorized into three classes: finite difference, binomial lattice methods, or Monte Carlo simulation.

Finite difference methods (Brennan and Schwartz, 1977) require establishing a differential equation to relate the option value to the stochastic variables, and then using discrete finite difference methods to numerically estimate the solution. The method is not very suitable for bi-level network design problems, where the differential equation would be difficult to specify.

Binomial lattice methods (Cox et al, 1979) assume that the probability distribution at each time state can be divided into two groups. By increasing the number of intervals up to the time horizon, it can be shown that the solution to the binomial tree converges to the actual solution. Trigeorgis (1991) developed a log-transformed version of the method that is consistent, numerically stable, and efficient. The log-transformed binomial lattice method could be used to obtain option values for multiple interacting options. His example illustrating the significance of properly evaluating multiple options is presented in Appendix A. This can have much significance in network-based options because many of the link component strategies can be viewed as individual options that would interact with each other similar to how multiple options actually do interact. FIGURE 2-6 illustrates a binomial lattice method.

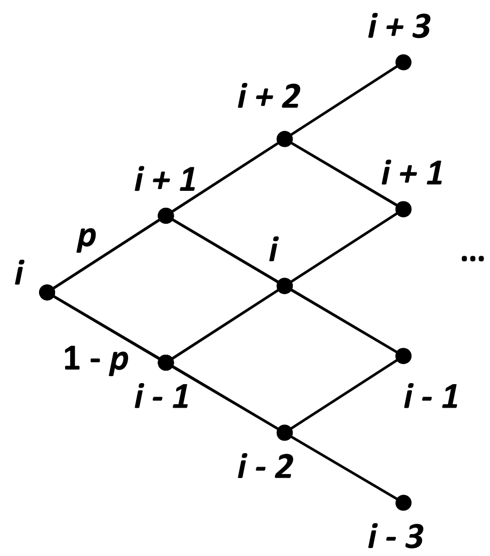

FIGURE 2-6. A binomial lattice type approach.

Despite the benefits of the method for dealing with multiple options simultaneously, it is not able to handle multi-dimensional variables very well because of its inherent method of simplifying probability distributions into branches. Some methods were developed as trinomial lattices, but the "curse of dimensionality" quickly becomes the dominant issue in the tractability of the method.

Gamba and Trigeorgis (2007) extend the original log-transformed binomial lattice method to multi-dimensional options. Compared to other efforts to achieve this same purpose, their proposed algorithm is also shown to be the most computationally efficient one. Earlier work produced the generalized log-transformed (GLT) approach for dimensions greater than 2, but they could result in negative probabilities for some parameter choices. The proposed adjusted GLT (AGLT) method transforms the position of the value by redefining it relative to the drift parameter.

The third type of method includes the Monte Carlo simulation methods (Boyle, 1977). The original methods involve simulation of multiple paths of realization for the stochastic process(es) and backwards dynamic programming. However, the simulation could get very computationally expensive because at any time step, the backwards dynamic program involves foreseeing the expected continuation function in the future time step.

This complication resulted in the Least Squares Monte Carlo simulation method (LSM) refined by Longstaff and Schwartz (2001). LSM reduces computational cost by using least squares regression at each intermediate time step based on the results of the following future time step along all the simulation paths. This idea is captured in FIGURE 2-7, where the Bellman equation (eq. (2.16) value $\Phi(t, p1)$ depends upon $E[\Phi(t+1)]$ obtained by least squares estimation from all the t+1 states where the current value is "in-the-money" (for maximizing a deferral option, this would mean having a positive NPV). The least squares fitting is performed with polynomial basis functions such as Laguerre or Hermite polynomials. In Chapter 3, the LSM method used to compute the deferral option value for the continuous network designs makes use of Hermite polynomials, although Laguerre polynomials can be used as well.

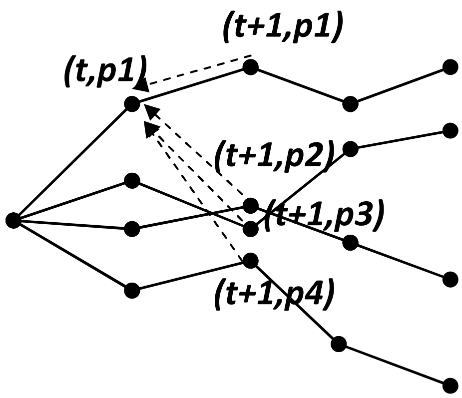

FIGURE 2-7. A Least Squares Monte Carlo simulation approach with 4 realized paths.

Compared to the earlier binomial lattice methods and Monte Carlo simulation, LSM is a very cost-effective method of obtaining option values for high-dimensional variables. The method has been proven to converge toward the actual option value as the number of paths, time steps, and number of polynomials for regression approach infinity. However, the method is known to have a downward bias for small samples. Another major disadvantage of the method is the inability to handle multiple interacting options. For the network-based options, overcoming this disadvantage would provide a method of decomposing the option values into link components.

Gamba (2002) overcomes this disadvantage for the LSM for three extreme cases: 1) the trivial case where the options are independent of each other, resulting in a summation; 2) purely compound options where one option depends on whether the prior option is exercised; and 3) mutually exclusive options where selecting one would eliminate the other alternative options.

2.2.6 Issues with Network-Based Options

Network-based strategies that rely on optimization models for design can also incorporate the optimization of timing at either 1) the link component level (link investment deferral option set); 2) the design level (network option design problem); or 3) separately as an investment (network investment deferral option). These approaches are discussed in much more detail in Chapter 3, which focuses on formulations as well as solution approaches with numerical examples. The AGLT binomial lattice method by Gamba and Trigeorgis (2007) offers a potential solution to computing the option values of the individual links treated as interacting options, but the multi-option LSM by Gamba (2002) offers a simpler solution that can be tailored to fit the algorithm to network option staging problems.

"It is easier to resist at the beginning than at the end." – Leonardo da Vinci

CHAPTER 3 NETWORK-BASED REAL OPTION MODELS

The literature review in Chapter 2 shows that network design models under uncertainty have been studied rather extensively, although very few have dealt with the complex nature of non-stationary stochastic processes. The few examples that were mentioned reside in the logistics literature such as Snyder et al's (2007) stochastic location model with risk pooling (SLMRP). However, these models do not incorporate congestion features in their formulations such as the bi-level problems facing urban transportation planners. Furthermore, the SLMRP model's decision variables are static in time; the problem is not designed to optimize an option to allocate investments to different network links over a maximum time horizon. Instead, the problem looks at obtaining the best allocation of resources for a present commitment.

Chapter 2 also reveals a growing literature in real option theory. Just as researchers in network design are grasping with methods to solving a dynamic decision-making problem under uncertainty, the world of corporate finance is finding ways of quantifying the value of flexibility in decision-making when thinking of it as an option with an expiration date.

Several researchers have applied real option methods to transportation investments, and some have looked at real option methods for supply chain network design (Huchzermeier and Cohen, 1996) or single period urban network toll design (Damnjanovic et al, 2008). However, no modeling framework and solution methodology based on real options has been proposed for multi-period urban network design problems under uncertainty.

Modeling the network design problem and solving it using conventional numerical methods will show that there can be significant value gained by decision-makers by being able to quantify the flexibility under uncertainty. Fairly new developments in real option solution methods such as those developed by Gamba (2002) enable the more complex problems to be solved.

This chapter presents three optimization models that incorporate flexibility into network design using real options. These models show the range of new strategies that a decision-maker can have when considering their network design projects as options. The three problems are solved for the Sioux Falls network and compared to illustrate that value to a decision-maker. Since the three models are specified for urban network design problems with bi-level formulations, there is a more generalized application of the models to many other network-related problems because they can be extended to simpler network design models as well.

3.1 MODEL FORMULATIONS

Starting with the network design formulations at the start of Chapter 2, there is a graph *G* of a set of nodes *N*, a set of links or arcs *A*, and a set of commodities *M*, subject to a set of constraints *S*. There is a continuous time element *t* and let's assume that the demand between each origin-destination pair and commodity, $q_t \in \mathbb{R}^{|N|^2|M|}$, evolves stochastically as discussed in Section 2.2.2. $|N|$ and $|M|$ are the number of nodes and number of commodities, respectively.

Other variables can also be considered stochastic variables, such as interest rate, degradable link capacities, or budget constraint, although for long term transportation planning purposes the OD demand generally exhibits the greatest uncertainty. For a multi-dimensional stochastic variable, the volatility parameter becomes a diffusion parameter that includes correlation between different OD pairs and commodities. An example is shown for a geometric Brownian motion characterization of the OD demand:

$$dq_t = \alpha \mathrm{I} q_t dt + \sigma q_t dW \quad (3.1)$$

where $\alpha \in \mathbb{R}^{|N|^2|M|}$ is the drift vector and $\sigma \in \mathbb{R}^{|N|^2|M|x|N|^2|M|}$ is the diffusion matrix. If OD pairs are independent of each other, then the diffusion matrix reduces to a volatility vector of size $|N|^2|M|$ times the identity matrix.

Other variables that need to be defined for these models are the initial budget *B*, a non-negative discount interest rate of ρ for all projects, and a planning horizon *T* after which a managing agent's option to invest in a project would expire. For realistic network design problems, there may be a subset of links $\bar{A} \subseteq A$ that can be invested upon.

3.1.1. Option Value of Flexible Network Design

The simplest approach to considering network design as a real option is to assume that the design solution is an investment and to compute a deferral option value given the set of link designs, similar to Huchzermeier and Cohen's (1996) treatment of real option value of supply chain designs, although multi-dimensional stochastic OD demand is considered here. Eq. (2.1) – (2.5) and Eq. (2.16) are combined to obtain the hierarchical network investment deferral option (NIDO) model. More specifically, Eq. (2.16) can be expressed explicitly in terms of the maximization of two time-dependent decision alternatives: defer or invest now in the optimal network design in that time. Additionally, for transportation planning purposes let's assume that compounding occurs at a discrete (e.g. annual) rate.

The result is a Bellman equation for determining the value of the option to invest in a network design as a function of stochastic OD demand.

$$\Phi_{t_n}(q_{t_n}) = \max\left\{\pi_{t_n}\left(\phi(q_{t_n}, y_{t_n})\right), (1+\rho)^{-\Delta t} E\left[\Phi_{t_{n+1}}(q_{t_{n+1}})\right]\right\} \quad (3.2)$$

where Δt is a discrete increment in time and t_n is a time state such that $t_{n+1} - t_n = \Delta t$ and $n = \frac{T}{\Delta t}$ is the final time horizon while $n = 0$ is the initial time state being solved for. ϕ is a system cost evaluation function such as total travel time that depends on the demand at time state t_n, q_{t_n}, and the optimal design y_{t_n} for minimizing the total cost. The payoff function $\pi_{t_n}: \mathbb{R} \to \mathbb{R}$ is the net present value of annuities minus the investment cost. Note that the option decision is not a function of the design of the network here; instead this modeling approach provides the value of flexibility to defer a network design in a time horizon with uncertainty in the demand.

The payoff value or NPV of annuities is based on the difference of the baseline total system cost with no investment, $\phi(q_{t_n}, 0)$, and the objective value with optimal network design found at time t_n, $\phi(q_{t_n}, y_{t_n}): \{\mathbb{R}^{|A|}, \mathbb{R}^{|\bar{A}|}\} \to \mathbb{R}$. The option value is a function of only the stochastic OD demand, $\Phi_{t_n}(q_{t_n})$.

The link design $y_{t_n} \in \mathbb{R}^{|\bar{A}|}$ and network design objective values $\phi(q_{t_n}, y_{t_n})$ are obtained by solving Eq. (2.1) – (2.5), where $q_{t_n} \in \mathbb{R}^{|N|^2|M|}$ is determined from the R_m for each origin-destination pair. This sub-problem is a mixed integer linear programming program that can be solved with standard network optimization methods depending on the type of problem. Further detail on solution methods are presented in the following Section 3.2.

3.1.1.1. *Variation 1: Congestion*

Several variations to this model exist. In an urban setting, congestion cannot be ignored and the generalized bi-level formulation from Eq. (2.7) – (2.10) replaces Eq. (2.1) – (2.5) in solving for the network design objective values. The implication of this substitution is that user equilibrium conditions for route choice are assumed and the solution methods would now require heuristics because bi-level network design problems are non-convex and non-differentiable. Because the NIDO model does not maximize the option value as a function of the design variables, existing solution heuristics for bi-level network design problems can be used with numerical methods for obtaining American-style option values.

3.1.1.2. *Variation 2: Passenger Travel Demand*

In passenger travel demand modeling, typically each origin-destination pair is treated as a single commodity. If the NIDO model is applied to this context, then the dimension of the stochastic demand variables would be reduced from $q_{t_n} \in \mathbb{R}^{|N|^2|M|}$ to $q_{t_n} \in \mathbb{R}^{|N|^2}$, where each commodity $m \in M$ is just an OD pair.

3.1.1.3. *Variation 3: Zero Drift*

In the case when the drift parameter is zero, the stochastic demand reduces to a martingale with a stationary expected value over time. This type of model can be used to address uncertainty similar to time-dependent scenario planning, where the expected demand is stationary but the uncertainty increases with the time horizon. Zero drift also benefits the solution method, as will be discussed in Section 3.2.

3.1.1.4. Variation 4: Continuous Network Design Problem

The network design model presented in the beginning of Chapter 2 is a generalized discrete network design problem. Many other network design problems exist as well, with a common alternative being the bi-level continuous network design problem (CNDP) and its variations (Yang and Bell, 1998). In the CNDP, the objective is to allocate budget to expand capacity at links in a network to minimize the total system cost. In this formulation, the Eq. (2.7) – (2.10) are modified from a mixed integer bi-level problem to a continuous variable bi-level problem.

$$\phi(q_{t_n}, y_{t_n}) = \min_y \sum_{a\in\bar{A}} c_a(y_{t_n,a})x_a + \sum_{a\in\bar{A}^c} c_a(0)x_a \tag{3.3}$$

Subject to

$$\sum_{a\in\bar{A}} d_a y_{t_n,a}^{\gamma} \leq B \tag{3.4}$$

$$0 \leq y_{t_n} \leq y_{max}, \quad y_{t_n}, y_{max} \in \mathbb{R}^{|\bar{A}|} \tag{3.5}$$

Where the link flows x_a are determined by the lower level user equilibrium program.

$$\min_x \sum_{a\in A} \int_0^{x_a} c_a(w, y_{t_n,a})dw \tag{3.6}$$

Subject to

$$\sum_k \zeta_k^{rs} = q_{t_n}^{rs}, \forall r, s \in N \tag{3.7}$$

$$\sum_r \sum_s \sum_k \zeta_k^{rs} \delta_{a,k}^{rs} = x_a \tag{3.8}$$

$$\zeta_k^{rs} \geq 0, \forall k \in K, \quad \forall r, s \in N \tag{3.9}$$

Where

γ is a parameter for relating the budget allocations to capacity improvements;

y_{max} is the maximum investment allocation allowed on link a;

ζ_k^{rs} is the flow on path $k \in K$ connecting origin r with destination s;

$\delta_k^{rs} \in \mathbb{R}^{|N|x|A|}$ is the node-link incidence matrix;

d_a is the cost of adding a unit improvement to link $a \in \bar{A}$;

$c_a(x_a, y_{t_n,a})$ is a travel cost as a function of investment $y_{t_n,a}$ and link flow x_a.

The construction cost vector $d \in \mathbb{R}^{|\bar{A}|}$ is assumed to discount at the same rate of interest as the budget, so that the same budget constraint in Eq. (3.4) can be used for all time periods.

The link cost function needs to be monotonically increasing, and a popular one created originally by the Bureau of Public Roads (BPR) is a power function shown in Eq. (3.10).

$$c_a(x_a, y_{t_n,a}) = \mathrm{c}_{\mathrm{a},0}\left(1 + \alpha\left(\frac{\mathrm{x_a}}{\mathrm{C_a} + \mathrm{y}_{\mathrm{t_n},\mathrm{a}}}\right)^{\beta}\right) \quad (3.10)$$

Where

α and β are parameters specific to the BPR function, typically 0.15 and 4, respectively;

$c_{a,0}$ is the free-flow travel cost on link a;

C_a is a capacity parameter.

In the CNDP, the link investments are continuous decision variables that increase the capacity parameter to reduce the cost of travel along a link. This problem has been shown to be non-convex and non-differentiable (Yang and Bell, 1998).

3.1.1.5. Variation 5: Fixed Design

The formulation of Eq. (3.2) has a subtle implication on a network design over time. At any future time state a managing agent that has not yet invested their budget has the option to consider investing in a new design solution given realizations in demand up to that time. This means that if time and new data reveals a dramatic shift in demand for some OD pairs and commodities, then the agent can choose a new design to adapt to this shift. This assumes that the cost of changing a design choice is negligible with respect to the budget.

NIDO can also be modified to obtain the option value for an agent who does not have the option to change their design in the future. This can be achieved by changing the y_{t_n} in Eq. (3.2) to y_{t_0}.

$$\Phi_{t_n}(q_{t_n}) = \max\left\{\pi_{t_n}\left(\phi(q_{t_n}, y_{t_0})\right), (1+\rho)^{-\Delta t} E\left[\Phi_{t_{n+1}}(q_{t_{n+1}})\right]\right\} \tag{3.11}$$

This changes the network design objective evaluations to $\phi(q_{t_n}, y_{t_0})$ for all time states. The result is that the agent loses the flexibility to re-design the network in the future because they have committed to an initial design. However, this restriction creates an easier problem to solve. It is possible to quantify the value of not committing to a preferred alternative in transportation planning by solving both Eq. (3.2) and (3.11).

3.1.1.6. Network Design Premium

To quantify the value of not committing to a preferred alternative, recall from Eq. (2.15) that the expanded NPV is the sum of the static NPV and an option premium. For a deferral option, the premium is the added value of the flexibility to defer an investment under uncertainty. For the NIDO model, this premium includes the flexibility to redesign the network as discussed in Section 3.1.1.5. This additional value due to design flexibility was introduced by Huchzermeier and Cohen (1996), and can be stated as the following observation: *The deferral option premium F of an investment over a network G(N,A) with M commodities is composed of the sum of a basic deferral premium with design commitment,* F_D*, and a non-negative flexible network design premium,* F_N*.*

Then the NIDO option premium *F* can be further decomposed into two premiums.

$$F = F_D + F_N \tag{3.12}$$

In other words, Eq. (2.15) can be expressed as $\Phi = NPV + F_D + F_N$ for network designs.

A direct conclusion can be obtained from the observation. Let $y_{t_n} \in \mathbb{R}^{|\bar{A}|}$ be a vector of link investment allocations at time state t_n. $F_N^{t_n}$, F^{t_n}, π_{t_n} and $\Phi_{\mathrm{t_n}}$ are the flexible network design premium, the option premium, the net present value of immediate investment, and the option value computed at time state t_n, respectively.

Proposition 3.1 *– For a given OD demand* $\sim GBM\left(\mu \in \mathbb{R}^{|N|^2|M|}, \sigma^2 \in \mathbb{R}^{|N|^2|M|x|N|^2|M|}\right)$*, a time horizon* T*, and discount rate* ρ*, a network investment* $y_{t_n} \in \mathbb{R}^{|\bar{A}|}$ *in* G(N,A) *with flexible network design premium* $F_N^{t_n}$ *= 0 and option premium* $F^{t_n} > 0$ *has an optimal design* y_{t_n} *for maximizing the option value* $\Phi_{\mathrm{t_n}}$*, whereas* $F_N^{t_n} > 0$ *is the opportunity cost of committing to the network design.*

Proof. Let's say that there's an investment with $F_N^{t_n}\left(y_{t_n}\right)$ = *0* and $F^{t_n}\left(y_{t_n}\right) > 0$, but there is another design $y_{t_n}^*$ such that $F_N^{t_n}\left(y_{t_n}^*\right) = 0$ and $\Phi_{\mathrm{t_n}}\left(\mathrm{y}_{\mathrm{t_n}}^*\right) > \Phi_{\mathrm{t_n}}\left(\mathrm{y}_{\mathrm{t_n}}\right)$, hence contradicting the design solution. It follows that there should be a number of future time states $t_{n+l}, l > 0$ such that $\pi_{t_{n+l}}\left(q_{t_{n+l}}, y_{t_n}^*\right)(1+\rho)^{-l\Delta t} > \pi_{t_n}\left(q_{t_n}, y_{t_n}\right)$. However, if such a solution exists, then at that time $t_{n+l} \leq t_{\frac{T}{\Delta t}}$ the optimal policy is to redesign from y_{t_n} to $y^*_{t_n}$, resulting in $F_N^{t_n}(y_{t_n}) > 0$. Hence it is contradictory and $F_N^{t_n} = 0$ if $F^{t_n} > 0$ and y_{t_n} is an optimal design.

If $F_N^{t_n} = 0$ and $F^{t_n} = 0$, the optimal decision is to either invest immediately if $\pi_{t_n} > 0$ or to reject the investment altogether. The problem is reduced to designing a y_{t_n} for a static NPV with the added constraint that $\pi_{t_{n+l}}(q_{t_{n+l}}, y_{t_{n+l}})(1+\rho)^{-l\Delta t} \leq \pi_{t_n}(q_{t_n}, y_{t_n})$ for all $n + l \leq \frac{T}{\Delta t}$ and $y_{t_{n+l}}$. Any feasible design solution found may not be a stochastic optimal with the highest expected static NPV, although the decision would be to invest (NPV > 0) or reject (NPV < 0) immediately because future conditions would be worse off.

If $F_N^{t_n} > 0$ and $F^{t_n} > 0$, then there are two possibilities due to the definition and non-negativity: $F_N^{t_n} = F^{t_n}$ or $F_N^{t_n} < F^{t_n}$. For the first case, the value from deferring is based purely on other designs, which means that the current design needs to be changed. In the latter case, there is value to waiting due to both the timing of the fixed design $F_D^{t_n}$ as well as the network redesign. Committing to the network design would forego the value of the flexible network design premium $F_N^{t_n}$ in favor of only the deferral premium $F_D^{t_n}$.

Two significant conclusions can be drawn from this network design premium. First, the traditional transportation planning practice is to commit to a preferred alternative and then to try and secure funding for it among a list of other competing projects in a regional transportation plan. Since these plans can sometimes take years to secure the funding, finalize design, and complete construction, it is prudent to incorporate flexibility into the planning process by monitoring these alternatives in relation to OD data and updating the alternatives as new data is obtained. In doing so, the priorities of these "conditional alternatives" can be determined from expanded NPV's that incorporate the value of deferral and design flexibility.

The second conclusion is that deferral options based on fixed designs can serve as lower bounds for flexible design options since the network design premium is non-negative. This is crucial for the following two models, which can be too complex to solve as flexible designs but are feasible as fixed design options.

3.1.2. Maximizing Option Value of Committed Network Design

The NIDO model computes the option value as a function of the optimal timing given an exogenous network design solution. Alternatively, the option value may be maximized as a function of both the initial network design and the timing decision. The problem becomes computationally intractable for the NIDO problem with the flexibility to re-design, because each time state would introduce another set of design variables and convergence cannot be achieved. However, this can be modeled if the fixed network design variation from Section 3.1.1.5 is considered. The *Network Option Design Problem* (NODP) maximizes the fixed design option value as a function of both a committed or fixed network design and the deferral decision. "Fixed design" and "committed design" will be used interchangeably but mean the same thing: a design that does not have the flexibility to be changed in the future but can still be deferred.

$$\Phi_{t_n}(q_{t_n}, y_{t_0}) = \max_{y_{t_0}} \left\{\pi_{t_n}\left(\phi(q_{t_n}, y_{t_0})\right), (1+\rho)^{-\Delta t} E\left[\Phi_{t_{n+1}}(q_{t_{n+1}}, y_{t_0})\right]\right\} \quad (3.13)$$

The difference between Eq. (3.13) and (3.2) is that the decision variables include the initial design. The consequence of this change is that now the heuristics for network design problems cannot be used directly. Instead, meta-heuristics such as genetic algorithm can be applied with option valuation numerical methods to obtain the solution. More details are provided in Section 3.2.

The model can be modified to handle congestion with bi-level network design problems for the payoff evaluation.

One key conclusion that can be drawn from the network design premium and Proposition 3.1 is that the fixed network design option Φ_D is always less than or equal to the flexible network design option Φ because $\Phi_D + F_N = \Phi$ and F_N is non-negative. NODP not only provides an optimal design for maximizing the fixed design option value; it also provides a lower bound to the value of the option under a flexible design setting.

3.1.3. Link Investment Deferral Option Set

For fixed discrete network designs, real options can be incorporated in yet another way. Suppose first that instead of individual links, groups of links can be considered as projects for computational efficiency. Instead of considering a set of links that can be invested, $\bar{A}$, groups of links can be considered as projects among a set of projects *L*. The network design can be treated as a set of interacting options as discussed in Chapter 2, where the decision to defer can be made to each link or project. The traditional design decision (whether to allocate budget to a link or not) is merged with the option decision (whether to invest in a link or to defer) into a multivariate set of decisions (which links in a committed design to invest immediately and which to defer).

Let's call this model the *Link Investment Deferral Option Set* (LIDOS). Unlike the NIDO and NODP Bellman equations, the objective of the LIDOS is to select which links to defer in order to maximize the incremental payoff in the current time state and the expected option value from the following time state. LIDOS is a stochastic project selection and staging model. This solution algorithm would be a backward dynamic program with forward elements, making it unsolvable using this approach.

Although the model cannot be solved directly, the key is to define it in such a way that a lower bound solution can be obtained. Extending Eq. (3.12) further, the option premium of this option set can be modeled as follows.

$$F = \sum_{l \in L}\left(F_{D,l} + F_{L,l}\right) + F_{LS} \quad (3.14)$$

Where $F_{D,l}$ is the deferral premium of an individual link, $F_{L,l}$ is the premium gained from the flexibility to invest in the remaining links in a fixed order, and F_{LS} is the premium from the flexibility of switching between one order and another. For example, a design with 3 discrete link investments would gain flexibility value by considering the individual deferral of the 1st, 2nd, and 3rd links. It would gain further value by allowing the network agent to switch the order from 1st, 2nd, and 3rd to 1st, 3rd, and 2nd, or any other permutation at a future time given that all of these links have been deferred.

Note that the network design premium F_N is not included. The backward dynamic programming objective is not compatible with the time-dependent network design constraints of the flexible design option such as a budget that would depend on the decisions from an earlier time state. Instead, a fixed initial design is not explicitly constrained in the formulation and can be used as the basis for staging interacting link options.

By defining the model's option premiums as the sum of deferral, flexibility of investing in remaining projects in an ordered combination, and the flexibility to change the order in the future, the model has been decomposed into non-negative portions where one portion is solvable. The solution methods described in Gamba (2002) are the closest to extending LSM to deal with multiple options with high-dimensional stochastic processes needed to solve this kind of problem. The extensions are designed for three extreme cases: a trivial case where the options are purely independent of each other, the purely compound case where each option is serially dependent upon the completion of the prior option, and the mutually exclusive case where choosing one option eliminates all others.

However, the LIDOS problem involves none of these three cases directly. Clearly the link investments cannot be treated as independent options because of network effects: investing in one link will impact the travel costs for all other links due to shifts in route choices. It is also not a case of mutually exclusive options since the LIDOS is defined such that all the link decision variables fit within the budget since they can all be invested on immediately as a discrete network design solution. The second stylized setting is to treat the link investments as an *ordered* set of compound options where investing in the $(j+1)^{th}$ option is only allowed if the j^{th} option is exercised in the ordered set h.

A simpler model and algorithm is proposed to handle only $F = \sum_{l \in L}(F_{D,l} + F_{L,l})$ without the option to change the order. Let's call this simpler model the *Ordered Link Investment Deferral Option Set* (OLIDOS) model. The advantage of this approach is that the complex problem of deferring individual link investments in any order can be bounded by a more simplified problem involving a fixed, ordered set of interacting options that can be solved by the multi-option LSM developed by Gamba (2002).

In Gamba (2002), this treatment of an ordered set of compound options can be modeled with the following Bellman equation:

$$\Phi_{h_j,t_n}(q_{t_n}) =$$

$$\max_{z_{h_j,t_n}} \left\{ \pi_{h_j,t_n}\left(\phi\left(q_{t_n}, z_{h_j,t_n}\right)\right) + \Phi_{h_{j+1},t_n}(q_{t_n}), (1+\rho)^{-\Delta t} E\left[\Phi_{h_j,t_{n+1}}(q_{t_{n+1}})\right]\right\} \quad (3.15)$$

Where the h_j is the j^{th} option among the set of L options in the ordered h^{th} set, $h \in H, H = |L|!$ different possible combinations of ordering the options. z_{h_j,t_n} is the set of first j options at time state t_n for ordered set h equal to 1. For example, TABLE 3-1 shows a sample portion of the 120 different combinations that exist for 5 interacting options. If the optimal decision for h = 1 is to invest in the first option immediately and defer the rest, then $z_{h_1,t_0} = 1$ and $z_{h_j,t_0} = 0$ for j = {2, 3, 4, 5}. For h = 1, h_1 = option 5.

TABLE 3-1. Ten of the 120 possible ordered sets for 5 options

Order set, *h*	h_1	h_2	h_3	h_4	h_5
1	5	4	3	2	1
2	5	4	3	1	2
3	5	4	2	3	1
4	5	4	2	1	3
5	5	4	1	2	3
6	5	4	1	3	2
7	5	3	4	2	1
8	5	3	4	1	2
9	5	3	2	4	1
10	5	3	2	1	4

Furthermore, Eq. (3.16) is proposed to select the ordered set that offers the maximum option value. The combination of Eq. (3.15) and (3.16) is the formulation for modeling OLIDOS.

$$\Phi_{t_n}(q_{t_n}, z_{t_n}) = \max_h\{\sum_j \Phi_{h_j,t_n}(q_{t_n})\} \qquad (3.16)$$

Among the three models, OLIDOS is the most versatile for managing agents because it provides decision-making insights at the individual link or project level. If all the projects are constrained to be invested together as one package, the model reduces to the fixed design option model. Solving the OLIDOS would not only provide the managing agent with a committed network design investment plan at the link level in the time dimension, it would also provide a lower bound on the value of deferring individual links of a flexible network design with flexible ordering of link investments.

FIGURE 3-1shows the comparison of the three models and how they relate to the Network Design Problem.

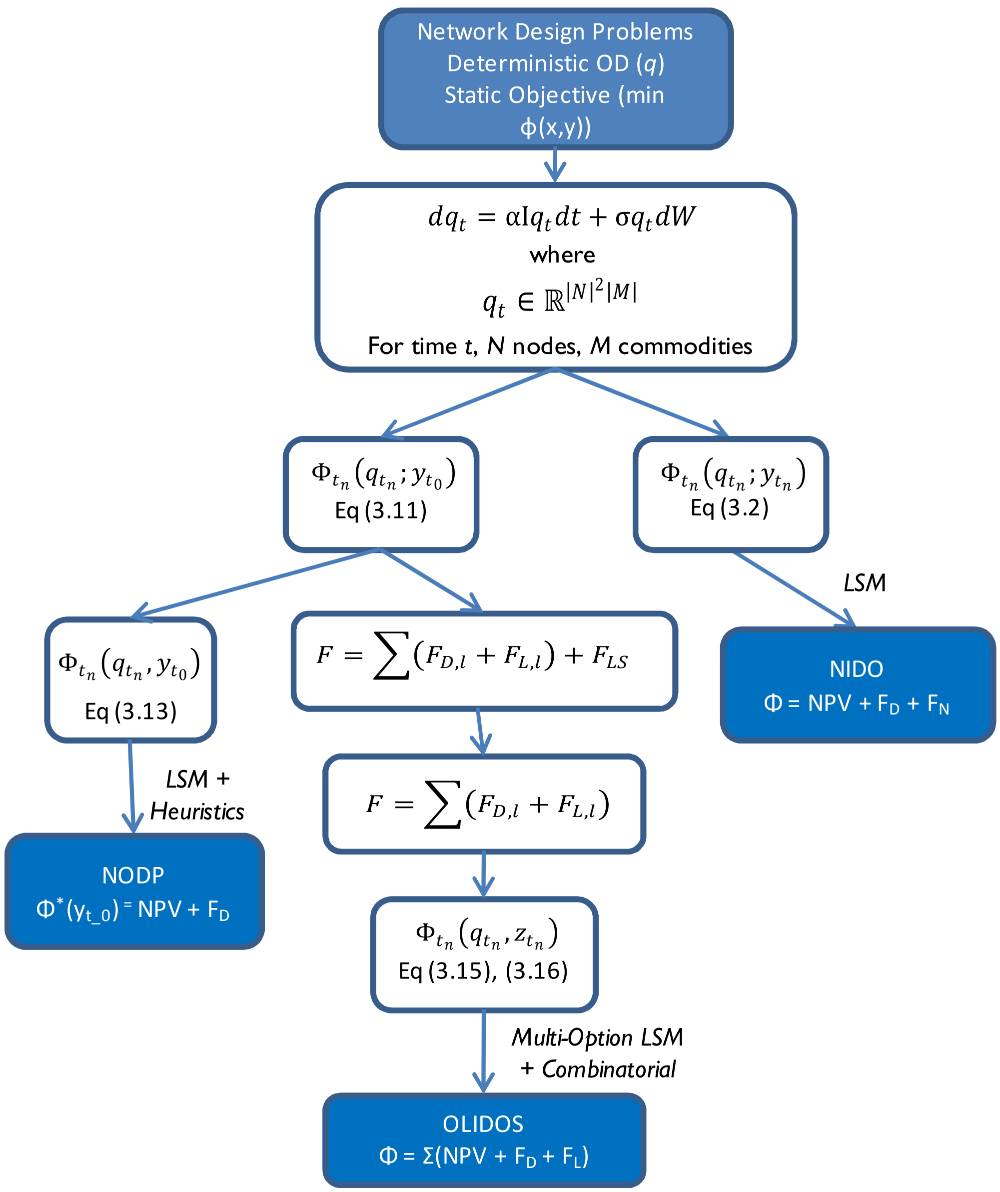

FIGURE 3-1. Comparison of Three Network-Based Real Option Models.

3.2. SOLUTION ALGORITHMS

3.2.1. Network Investment Deferral Option

NIDO is a single option dynamic programming problem with a network-based objective function $\phi(q_{t_n}, y_{t_n})$ that depends on multi-dimensional stochastic variables q_{t_n}. Because of the complex nature of some network design problems, it is not suitable to use finite difference methods to solve them. Because network optimization problems can have high computational cost, a relatively fast converging numerical option valuation method is necessary. This requirement rules out traditional Monte Carlo simulation methods as well. The multi-dimensionality of the stochastic variables makes it difficult to apply traditional binomial lattice methods to solve the problem.

The Least Squares Monte Carlo (LSM) method is chosen for solving the NIDO value. The method provides a pathwise approximation to the optimal stopping rule for maximizing the value of an American option. In order to apply this method, the q_{t_n} needs to be in a discrete format and certain assumptions need to be made for the payoff function to obtain the NPV.

3.2.1.1. Demand Simulation

The OD demand simulation can be derived from Eq. (3.1) and Section 2.2.2. In general, correlated demand can be simulated by applying Cholesky decomposition methods and simulating the processes as independent variables. Because of that, the notation is for one-dimensional elements of multi-dimensional vector parameters.

$$q_t^{rs} = q_{t-\Delta t}^{rs} \exp\left(\left(\mu_{rs} - \frac{\sigma_{rs}^2}{2}\right)\Delta t + \sigma_{rs} W_{\Delta t}\right) \quad (3.17)$$

Where $\alpha \in \mathbb{R}^{|N^2||M|}$ is a generalized drift parameter and $\sigma \in \mathbb{R}^{|N^2||M|x|N^2||M|}$ is a generalized diffusion parameter, and $W_{\Delta t}$ can be simulated with a normal inverse function with $\varepsilon\sqrt{\Delta t}$, where $\varepsilon \sim N(0,1)$. If the OD demands are independent of each other, the σ becomes a diagonal matrix. Time series data of OD demand would be necessary to estimate the parameters.

3.2.1.2. Payoff Function Valuation

Before determining the net present value, the network design objective value $\phi\left(q_{t_n}, y_{t_n}\right)$ needs to be solved using network design algorithms. For example, a minimum spanning tree problem might utilize a greedy algorithm. On the other hand, a continuous network design problem would require bi-level heuristics.

The method of obtaining the net present value depends on the drift parameter of the stochastic variable(s) and whether the network design problem is bi-level with congestion. If the drift parameters are zero, i.e. the expected value remains the same, then the net present value of an infinite series of equal payments with constant interest rate would be:

$$\pi_t(\phi(q_t, y_t); \mu = 0) = \frac{\lambda\left(\phi(q_t, 0) - \phi(q_t, y_t)\right)}{1-(1+\rho)^{-1}} - \sum_{a \in \bar{A}} d_a y_{t,a}^{\gamma} \quad (3.18)$$

The first term on the right-hand side is the net present value of infinite annual returns. The second term is the investment cost.

For non-zero drift values, an incremental assignment heuristic is proposed. For each increasing Δt, the expected value of all OD increments Δq_t are assigned to the network using the shortest paths based on link travel times corresponding to the updated values from the previous time increment.

Algorithm 3.1

1. Let $\pi_{t_n}\left(\phi\left(q_{t_n}, y_{t_n}\right)\right) = \lambda\left(\phi\left(q_{t_n}, 0\right) - \phi\left(q_{t_n}, y_{t_n}\right)\right) - \sum_{a \in \bar{A}} d_a y_{t_n,a}^{\gamma}$.
2. Let $l = 1$.
3. If there's congestion, update the link costs $c_{t_{n+l}} = c_{t_{n+l}}(x_{t_{n+l-1}})$.
4. Let $\pi_{t_n} = \pi_{t_n} + (1+\rho)^{-i\Delta t}\lambda\left(\phi\left(E\left[q_{t_{n+l}} - q_{t_{n+l-1}}\right], 0\right) - \phi\left(E\left[q_{t_{n+l}} - q_{t_{n+l-1}}\right], y_{t_n}\right)\right)$ using $c_{t_{n+l}}$ for the link cost functions and all-or-nothing shortest path assignment.
5. If the change in π_{t_n} is greater than some tolerance, let $i = i + 1$ and go to step 2, else end.

Step 3 assigns the incremental expected flow onto the updated network, evaluates the benefit between having an investment against a baseline with no investment, converts it to an annual increment, and discounts it back to the t_n time state. If there's no congestion, the cost function does not need to be updated.

Convergence of this heuristic is not guaranteed if the absolute value of drift rates are so large that the incremental change in net present value from all-or-nothing assigned flow differences exceeds the discounting interest rate. The threshold would depend on the current congestion and capacity levels in the network, as applicable. Because of this, NIDO valuation with bi-level network design problems having non-zero drift rates should be tested for convergence.

3.2.1.3. Solution Algorithm

The LSM algorithm with network payoff valuations is shown below for a given network $G(N,A)$, OD demand q_{t_n} with estimated parameters μ and σ for GBM process, budget B, interest rate ρ, time horizon T, and a number of simulation paths P.

Algorithm 3.2

1. For each path $\omega \in P$ and time state $t_n, 0 \leq n \leq \frac{T}{\Delta t}$, generate simulated OD demand using Eq. (3.17).
2. For the known q_{t_0} and for each of the simulated $q_{t_n}(\omega), 0 \leq n \leq \frac{T}{\Delta t}, \omega \in P$, evaluate a network design objective such as Eq. (3.3) – (3.9) to obtain y_{t_n} and use Eq. (3.18) or Algorithm 3.1 to obtain $\pi_{t_n}\left(\phi\left(q_{t_n}, y_{t_n}\right)\right)$.
3. Starting from $n = \frac{T}{\Delta t}$, use LSM to solve the backwards dynamic program Eq. (3.2).

a. If $n = \frac{T}{\Delta t}$, let $\Phi_{t_n}\left(q_{t_n}(\omega)\right) = \max\left(0, \pi_{t_n}\left(\phi(q_{t_n}, y_{t_n})\right)\right)$.

b. Keep track of the optimal decision with the variable $\theta(\omega, t_n)$; if $\Phi_{t_n}\left(q_{t_n}(\omega)\right) > 0$, then that sample path is "in the money" and it is assigned a value of $\theta(\omega, t_n) = 1$. Otherwise, let $\theta(\omega, t_n) = 0$.

c. Let $n = n - 1$. If $n = 0$, go to step 4.

d. Estimate $\widehat{\Phi}_{t_{n+1}}$ using least squares regression with Hermite polynomial series, although other polynomials that form an orthonormal basis such as Laguerre series can also be used:

$$H_i(x) = (-1)^i e^{\frac{x^2}{2}} \frac{d^i}{dx^i} e^{-\frac{x^2}{2}} \tag{3.19}$$

Where the first four polynomials in the series are:

$H_0(x) = 1$

$H_1(x) = x$

$H_2(x) = x^2 - 1$

$H_3(x) = x^3 - 3x$

The $(i+1)^{th}$ polynomial can be represented recursively as:

$$H_{i+1}(x) = xH_i(x) - iH_{i-1}(x) \tag{3.20}$$

A regression function with *Π* polynomials could be of the form:

$$\Phi_{t_{n+1}} = \sum_{i=0}^{\Pi} \beta_i (-1)^i e^{\frac{x^2}{2}} \frac{d^i}{dx^i} e^{-\frac{x^2}{2}} \tag{3.21}$$

Where the β_i coefficients are estimated using least squares. The *x* values are $\pi_{t_n}\left(\phi\left(q_{t_n}(\omega), y_{t_n}\right)\right)\Big|_{\pi_{t_n}>0}$ at each simulation path at the particular time interval.

e. Use the estimate $\widehat{\Phi}_{t_{n+1}}$ to solve Eq. (3.2). If the optimal decision is to wait because $(1+\rho)^{-\Delta t} E\left[\widehat{\Phi}_{t_{n+1}}\right] > \pi_{t_n}\left(\phi\left(q_{t_n}(\omega), y_{t_n}\right)\right)$, then the option on that simulation path is still in the money and $\theta(\omega, t_n) = 1$.

f. If $n > 0$, go to step c. Otherwise, the value obtained from Eq. (3.2) is the value of the option.

4. If $\Phi_{t_0} > 0$, then the option is worth keeping. If $(1+\rho)^{-\Delta t} E\left[\widehat{\Phi}_{t_1}\right] > \pi_{t_0}\left(\phi\left(q_{t_0}, y_{t_0}\right)\right) > 0$, then the best strategy is to defer the option. If $\Phi_{t_0} > 0$ and $(1+\rho)^{-\Delta t} E\left[\widehat{\Phi}_{t_1}\right] \leq \pi_{t_0}\left(\phi\left(q_{t_0}, y_{t_0}\right)\right)$ then the best strategy is to invest immediately.
5. The algorithm may need to be re-run at increasing values of *Π* until the option value stops increasing beyond some tolerance.

3.2.1.4. Convergence

Proofs of convergence are available from Longstaff and Schwartz (2001) for the LSM algorithm for both the continuous time American options as well as the discrete time options. The theorem from Longstaff and Schwartz is re-stated here in common notation for the discrete time options while their proof is provided in Appendix B.

Theorem 3.1 (Longstaff and Schwartz, 2001) – *Assume that the value of an American option depends on a single state variable X with support on (0,∞) which follows a Markov process. Assume further that the option can only be exercised at times* t_1 *and* t_2*, and that the conditional expectation function* $\Phi(\omega; t_1)$ *is absolutely continuous and*

$$\int_0^\infty e^{-X}\Phi^2(\omega; t_1)dX < \infty$$

$$\int_0^\infty e^{-X}\Phi_X^2(\omega; t_1)dX < \infty$$

Then for any $\epsilon > 0$*, there exists an* $\Pi < \infty$ *such that*

$$\lim_{P\to\infty} \Pr\left[\left|\pi(X) - \frac{1}{P}\sum_{i=1}^{P} LSM\left(\omega_i; \Pi, \frac{T}{\Delta t}\right)\right| > \epsilon\right] = 0$$

The conclusion is that selecting *Π* large enough and letting $\frac{T}{\Delta t} \to \infty$ would result in an option value that is within ϵ of the true value. The theorem is limited to one-dimensional variables, although Longstaff and Schwartz noted that it should be applicable to multi-dimensional variables where uniform convergence occurs.

Scaling issues may arise when using LSM with high *Π*. In these cases, the resulting option value will be clearly divergent. The least squares regression can be scaled by the original NPV value to reduce matrix singularity in the estimation process.

3.2.1.5. Solving Variation 1: Congestion

The solution Algorithm 3.2 can be used to solve the five variations presented in Section 3.1. In particular, the urban road design problem with congestion effects requires the use of heuristics to obtain the network design solution for payoff valuation. Yang and Bell (1998) presents several solution methods to these types of problems, including the simple Iterative Optimization Algorithm (IOA) originally developed by Steenbrink (1974); Link Usage Proportion-based (LUPB) algorithms; Sensitivity Analysis Based (SAB) methods; and branch and bound methods for discrete network design problems first applied by LeBlanc (1975).

Global stochastic search methods have also been developed for congestion effects, such as Friesz et al's (1992) simulated annealing approach with variational inequality constraints and the response surface approach in Chapter 5 for continuous network design problems.

3.2.1.6. Solving Variation 4: Continuous Network Design Problem

While there are better performing heuristics among those listed in Section 3.2.1.5, the IOA heuristic is a computationally cheap approach that is crucial to evaluating the option value with LSM. IOA involves iteratively fixing one of the decision variables at one level to solve for the other decision variable, and then alternating to the other level with the updated solution. For example, a bi-level problem with capacity expansion would be solved by fixing user equilibrium flow rates with no investments allocated, solving for the optimal investment allocations as a single level program, and then re-solving the lower level user equilibrium problem with updated investment allocations.

Steenbrink showed that when ignoring Braess' paradox, the algorithm converges to a local optimum, which Yang and Bell (1998) noted is the Cournot-Nash equilibrium rather than a more appropriate Stackelberg equilibrium.

3.2.1.7. Solving Variation 5: Fixed Design

Algorithm 3.2 can be used to solve the fixed design option value by simply obtaining the payoff value in Eq. (3.2) with the initial network design solution instead of applying IOA in each simulation path state. In other words, the user equilibrium at each simulation path needs to be computed with the investment allocation from the initial time y_{t_0} instead of solving the optimal allocations at time state t_n.

The y_{t_0} variable represents the set of allocations made at the initial time. The NIDO model reduces from a joint option to defer and redesign to one that can only defer implementation of the initial design.

3.2.1.8. Obtaining the Flexible Network Design Premium

To obtain the network design premium, note that the static NPV is the same at the base year for both the NIDO and the fixed design NIDO. Hence, the design premium is simply the difference of the NIDO option premium and the fixed design premium.

3.2.2. Network Option Design Problem

Maximizing the fixed design option value $\Phi_{t_n}(q_{t_n}, y_{t_n})$ taking into account the whole time horizon *T* can be accomplished because there are no decision variables except in the base time t_0. Fast converging global heuristics are necessary because of the computational cost of one evaluation function, which is $O\left(P * \frac{T}{\Delta t} * UE(N)\right)$, where *P* is the number of simulation paths, *T/Δt* is the number of time states, and UE(N) is the computational cost of one user equilibrium assignment or network optimization which scales with the size of the network.

A heuristic for solving the NODP with continuous network design problem is demonstrated, leading to the continuous network option design problem (CNODP). A radial basis function approach is adopted to solve this problem. Chapter 5 deals in detail with this approach for solving the CNDP, so only a brief introduction is provided here.

Chapter 5 introduces a solution method for continuous network design problems based on the Metric Stochastic Response Surface (MSRS) method developed by Regis and Shoemaker (2007). MSRS is a global stochastic optimization approach that can use radial basis functions (RBF's) to intelligently guess the next point to evaluate using interpolation (MSRBF). Their multi-start local MSRBF algorithm is shown to work extremely well for continuous, high-dimensional, multimodal, computationally expensive functions with box constraints. Most importantly, it has been shown to converge faster than other non-derivative based heuristics such as genetic algorithm and simulated annealing, for functions with up to 14 dimensional variables. In chapter 5 we show that the method performs significantly better than the genetic algorithm for network design problems with up to 31 dimensional variables for the Anaheim, CA network.

Using this approach, the solution algorithm would involve guessing an initial set of design solutions, solving the fixed design option value using the approach discussed in Section 3.2.1.7 for each element of the set, and then proceeding with the RBFCNDL algorithm discussed in Chapter 5.

Algorithm 3.3

1. Generate initial set of solutions as per RBFCNDL algorithm in Section 5.2.2.
2. For each solution, apply Algorithm 3.2 to evaluate the fixed design option value, $\Phi_{t_n}\left(q_{t_n}, y_{t_0}\right)$.
3. Continue the RBFCNDL algorithm until the N_{max} number of iterations are reached or a number of multiple local starts have converged.

The solution is a lower bound for a flexible network design deferral option where future design decisions can be changed.

3.2.2.1. Convergence

The fixed design option value exists and can be found for a given network design. The RBFCNDL algorithm requires the decision variables to be continuous variables, which they are for the CNODP. Based on the convergence criteria of Regis and Shoemaker

(2007), this algorithm would probabilistically converge as the maximum number of iterations $N_{max} \to \infty$. The proof of convergence is provided in Appendix C.

3.2.3. Ordered Link Investment Deferral Option Set

The approach to solving this multi-option LSM from Gamba (2002) is to repeat the LSM from Longstaff and Schwartz (2001) from the last option in the ordered set *h* to the first option in the ordered set for each time state and simulation path. Each ordered set is enumerated to obtain the exact multi-option LSM solution value.

Algorithm 3.4

1. Obtain $y^*_{t_0} \in \mathbb{R}^{|\bar{A}|}$ using a discrete network design solution, or creating a set of discrete projects $z^*_{t_0} \in \mathbb{R}^{|L|}$ from a continuous network design solution, where $z^*_{t_0} = \{z^*_{t_0,1}, \dots, z^*_{t_0,|L|}\}$ and $z^*_{t_0,i} = \sum_{a \in L_j} y^*_{t_0,a}$ where L_j is a predefined j^{th} project among $|L|$ projects and $\{\cup\, a \in \bar{A}\} = \{\cup_{j \in |L|}\{\cup\, a \in L_j\}\}$.
2. For those links that are being considered for investment, determine the number of different combinations for ordering them. Let $H = |L|!$.
3. For each ordered set $h \in H$, solve Eq. (3.15) using Gamba's multi-option LSM method. It should be the same as Algorithm 3.2, plus an additional loop at each time state for a sequenced evaluation of each j^{th} option, going backwards from the last option in the ordered set *h*.
    a. For networks, a cumulative computation of the payoff for each additional link or project needs to be made for every time state t_n and simulation

path ω. For 5 projects, that means determining the system cost for the 1st project at t_n and ω, then for the first two projects, then first three projects, etc.

b. The payoff value of the j^{th} project is the difference $\sum_{l=1}^{j} \pi_{h_l,t_n} - \sum_{l=1}^{j-1} \pi_{h_l,t_n}$.

c. Project options with payoffs that are less than zero are not treated as zero because they can contribute to the prior project option's value; instead, if $\pi_{h_j,t_n}\left(\phi\left(q_{t_n}, z_{h_j,t_n}\right)\right) < 0$ let $\pi_{h_j,t_n}\left(\phi\left(q_{t_n}, z_{h_j,t_n}\right)\right) = \pi_{h_j,t_{n+1}}\left(\phi\left(q_{t_{n+1}}, z_{h_j,t_{n+1}}\right)\right)(1+\rho)^{-1}$.

d. When evaluating the decision rule, when $\pi_{h_j,t_n}\left(\phi\left(q_{t_n}, z_{h_j,t_n}\right)\right) + \Phi_{h_{j+1},t_n}\left(q_{t_n}\right) < (1+\rho)^{-\Delta t} E\left[\Phi_{h_j,t_{n+1}}\left(q_{t_{n+1}}\right)\right]$, the current option and all following options would have to be set to defer.

e. Set $\theta_{h_j}(\omega, t_n) = 1$ for any pathwise decision to invest immediately at time t_n.

f. When the initial year is reached, for each j^{th} option discount back the option value where the immediate investment decisions were made for all paths in P and take the average: $E\left[\Phi_{h_j,t_1}\left(q_{t_1}(\omega)\right) \middle| \theta_{h_j}(\omega, t_n) = 1\right] = \frac{\sum_P \Phi_{h_j,t_n}\left(q_{t_n}(\omega)\right)(1+\rho)^{-n}}{P}$.

g. Going backward from the final option, let:

$\Phi_{h_j,t_0} = \max\left(\pi_{h_j,t_0}, E\left[\Phi_{h_j,t_1}\left(q_{t_1}(\omega)\right) \middle| \theta_{h_j}(\omega, t_n) = 1\right]\right)$.

4. The option value is the maximum of the sum of option values obtained from each ordered set *h*, the solution to Eq. (3.16).

The solution is a lower bound for a flexible network design deferral option where future design decisions can be changed.

3.2.3.1. Convergence and Computational Cost

Since the algorithm is just an extension of LSM, the same convergence criteria apply. However, the multi-option LSM needs to be evaluated for each option in an ordered set and the option value needs to be evaluated for each ordered set in *H*. Although $H = |L|!$, some of the combinations are repeated so that one set of ordered jth option of *h* = 1 at time t_n can also be re-used for another ordered set *h* = 2. This effectively makes the computational cost on the order of $O\left(\sum_{j=1}^{|L|} \frac{|L|!}{j!(|L|-j)!} * P * \frac{T}{\Delta t} * UE(N)\right)$. For 5 sets of links and 5 time periods with 300 sample paths, this is equivalent to *O*((5+10+10+5+1)*300*5**UE*(*N*)) = *O*(46,500**UE*(*N*)). As a combinatorial problem there is still a challenge in solving this problem for large number of sets of links, and future research should address this need for larger network design considerations. For example, if all ten links in the typical Sioux Falls example are treated as separate options, this could lead to a computational time of *O*(2046*300*5**UE*(*N*)), which is 33 times the computational cost from doubling the number of links from five to ten.

A possible method to address this combinatorial problem may involve using meta-heuristics or a two-step approach: the first step would involve a smaller number of sample paths as a sample to identify potential candidates, and then using the desired number of sample paths with only the candidates for a more rigorous solution in a second stage. This method will be explored in future research.

3.3. NUMERICAL EXAMPLE

The real option analysis is applied to a common network example to provide a platform for comparing its results. The Sioux Falls network as described, is referenced in Chen and Yang (2004) and Suwansirikul (1987), but is the classic example used by researchers in this field. For specific details of the Sioux Falls network parameters used here and for future chapters, please refer to Appendix D.

The solution to the standard CNDP with the baseline demand flows is obtained using IOA to compare with the performance of previous work done on Sioux Falls. The d_a for the link improvements are shown in Appendix D. A value of total system travel time (TSTT) = 75.942 was obtained here using a maximum of 100 iterations of Frank-Wolfe and 1% objective value tolerance. This is in comparison to a no investment user-optimal cost of 101.171.

3.3.1. Network Investment Deferral Option

To test the NIDO model on the Sioux Falls network, consider the case where each of the 552 OD pairs evolves independently as a geometric Brownian motion. For simplicity, let's assume that the drift parameter μ_{rs} = 0 for all OD pairs *rs*, and let's consider continuous link expansions as the design variables. Since we are looking at a multi-year setting with discounting factor and a stationary mean, we want to choose a conversion rate λ which would have a 1 VHT to \$1000K NPV translation so that a budget of \$5.5M would still equal to 5.5 VHT cost. Let

$$\lambda = 1000/\left(\frac{1}{1-\frac{1}{1.06}}\right) = 56.604$$

The net present value of an immediate investment is:

$$NPV = (101.171 - 75.942)(56.604) - 5500 = 19{,}730 = \$19.73M$$

Under these deterministic settings, the positive NPV suggests that the decision-maker should invest in the network design immediately for a net payoff value of \$19.73M.

3.3.1.1. Number of Sample Paths

Suppose instead that there is non-stationary uncertainty introduced into the OD demand. Let the volatility of each OD pair be σ_{rs} = 0.35. This means that the demand for each OD pair is log-normally distributed such that $\ln\left(\frac{q_{t+\Delta t}^{rs}}{q_t^{rs}}\right) \sim N\left(q_t^{rs}, 0.35\sqrt{\Delta t}\right)$. As the length of time increases, the uncertainty in the forecast of the demand increases.

One run of the LSM simulation algorithm with P = 300 sample paths and *Π* = 6 basis functions results in a distribution of options exercised shown in TABLE 3-2. This result indicates that almost 70 percent of the time the optimal decision to exercise the 5-year investment option would lie within the first year of realization.

TABLE 3-2. Sample Distribution of Simulated Option Exercises for T =5, P = 300, *Π* = 6, σ_{rs} = 0.35

Time (yrs)	Frequency	Percent
1	203	67.7%
2	54	18.0%
3	34	11.3%
4	5	1.7%
5	4	1.3%

Thirty runs of the LSM algorithm for the NIDO value are conducted with *P* = 30 and *P* = 300 sample paths to compare the variance as a function of sample paths. The LSM algorithm uses the same *Π* = 6 basis functions determined as a reasonable number with a maximum obtained option value and minimum variance. A five year deferral horizon similar to a five year TIP program is used with a 6% interest rate to reflect realistic transportation planning settings. The results of the 60 total runs are plotted in FIGURE 3-2. The standard error decreases from 3.1% of the total expected value ($0.77M) at *P* = 30 to 1.9% ($0.42M) at *P* = 300 sample paths. The actual results for *Π* = 3 to 6 are included in Appendix E.

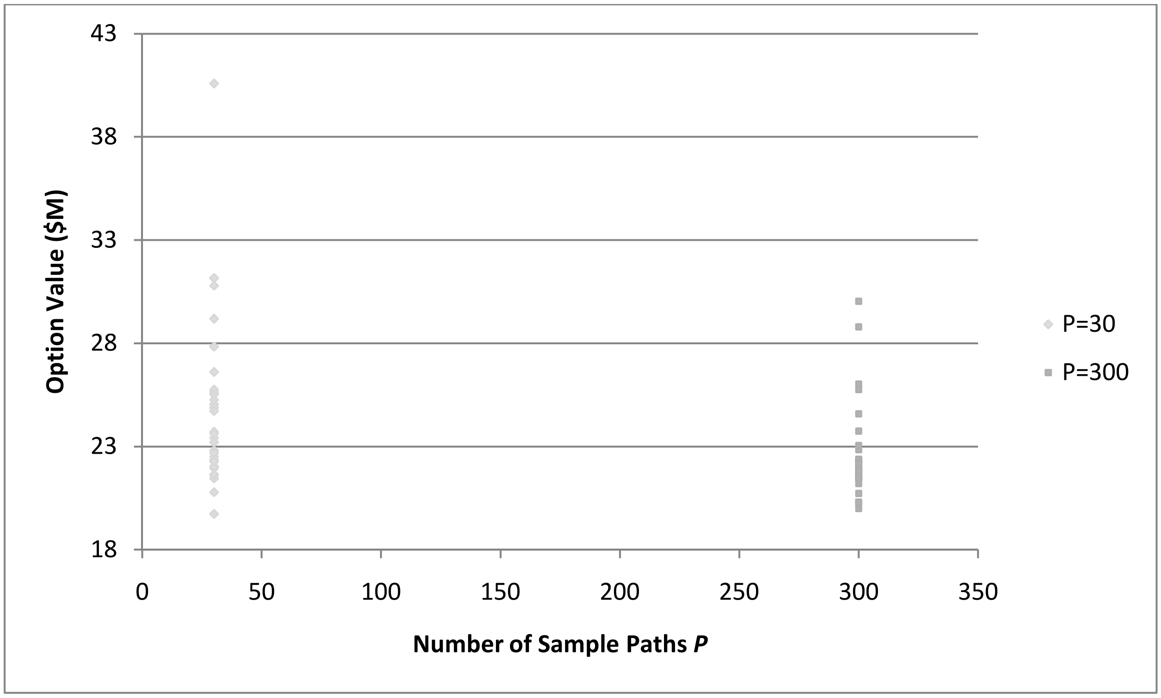

FIGURE 3-2. Option value from multiple runs at *P* = 30 and *P* = 300 sample paths with μ = 0 and σ = 0.35, using Π = 6 basis functions for LSM.

Several conclusions can be drawn from this first test.

First, the test shows that the algorithm converges for *P* = 300 sample paths for the example using a relative standard error of 2% as the tolerance.

Second, the mean of the NIDO value at *P* = 300 is $22.6M, compared to the fixed design option value of $21.4M. In other words, at a volatility of 0.35, the optimal decision is to defer whether or not there is flexibility to re-design the network, but keeping the flexibility would increase the value of the investment by $1.2M. This is the value of the option to redesign the network. If redesigning a network in the future would cost at least an additional $1.2M, then it is not worth it to keep this option open. This is the value of maintaining conditional alternatives instead of a single preferred alternative.

Lastly, this option makes use of the deferral strategy in investment but other simple option strategies can be substituted in. For example, a long term construction project with high demand volatility may consider abandonment.

3.3.1.2. Volatility

A test is also conducted of the option values as functions of the homogeneous OD demand volatility parameters. As expected, the fixed design deferral option value would increase as the volatility increases, just as the NIDO with the network design premium would increase. The volatility threshold lies somewhere near 0.30 for this particular example, as the optimal decision is to invest immediately if $\sigma_{rs} < 0.30$ and to defer otherwise.

The vertical gap between the NIDO value curve and the fixed design deferral value curve is the value of the network design premium. This value appears to increase as volatility increases as well, which means that the NIDO value increases at a faster rate than the deferral option alone. This makes sense since greater volatility would lead to a wider range of possible scenarios so the possibility for the current design to work just as well for all scenarios would diminish, ultimately resulting in a higher value to redesign.

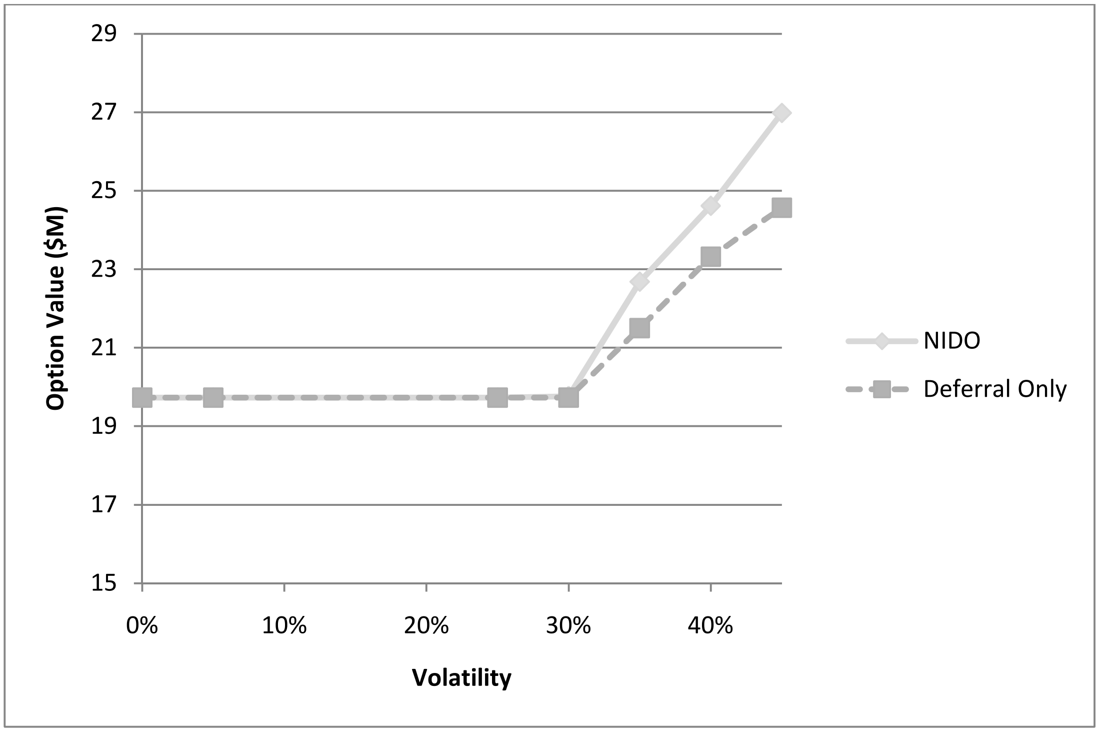

FIGURE 3-3. Option value as a sum of NPV, fixed design deferral (F_D), and network design premium (F_N) as a function of volatility.

At 0.35 volatility, the fixed design option value from the single run is $21.50M while the NIDO option value is $22.68M.

3.3.1.3. Time Horizon

The NIDO and fixed design deferral option values are plotted against a varying time horizon up to 20 years to show that the value increases as the time until option expiration is increased. The same random seeds used to simulate the 0.35 volatility in Section 3.3.1.2 are expanded to 20 years here to have a common starting point.

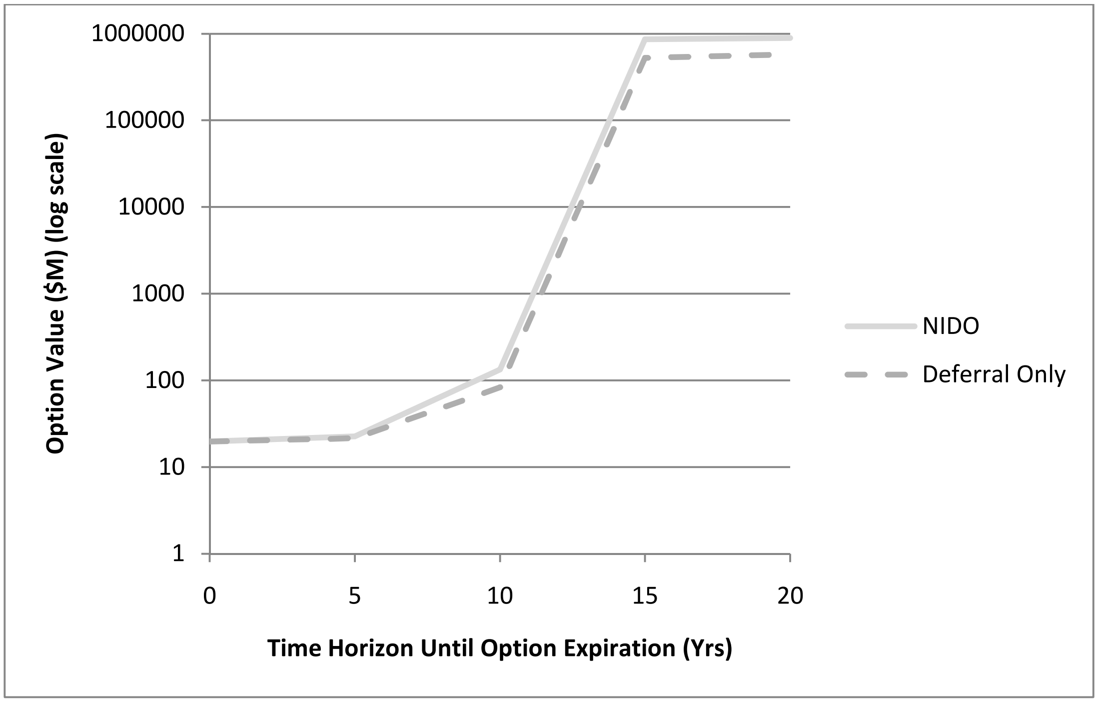

FIGURE 3-4. Option value as a sum of NPV, fixed design deferral (F_D), and network design premium (F_N) as a function of volatility.

From 5 to 15 years, the option value appears to increase exponentially from $22.68M, so a logarithmic scale is used for the figure. Between 15 and 20 years, the option value appears to have reached a steady state value due to the interaction of the 6% interest rate and the volatility of the demand flows with zero drift. The results suggest that $900B is a steady state solution for an infinite time horizon that can be reached for an expiration period of 15 years or more.

3.3.2. Network Option Design Problem

As shown in FIGURE 3-5, Algorithm 3.3 is run for two local starts resulting in 266 option evaluations for Sioux Falls with 35% volatility, T = 5 years, P = 300, Π = 6, and using the same random seed as Section 3.3.1.2 for the OD simulation.

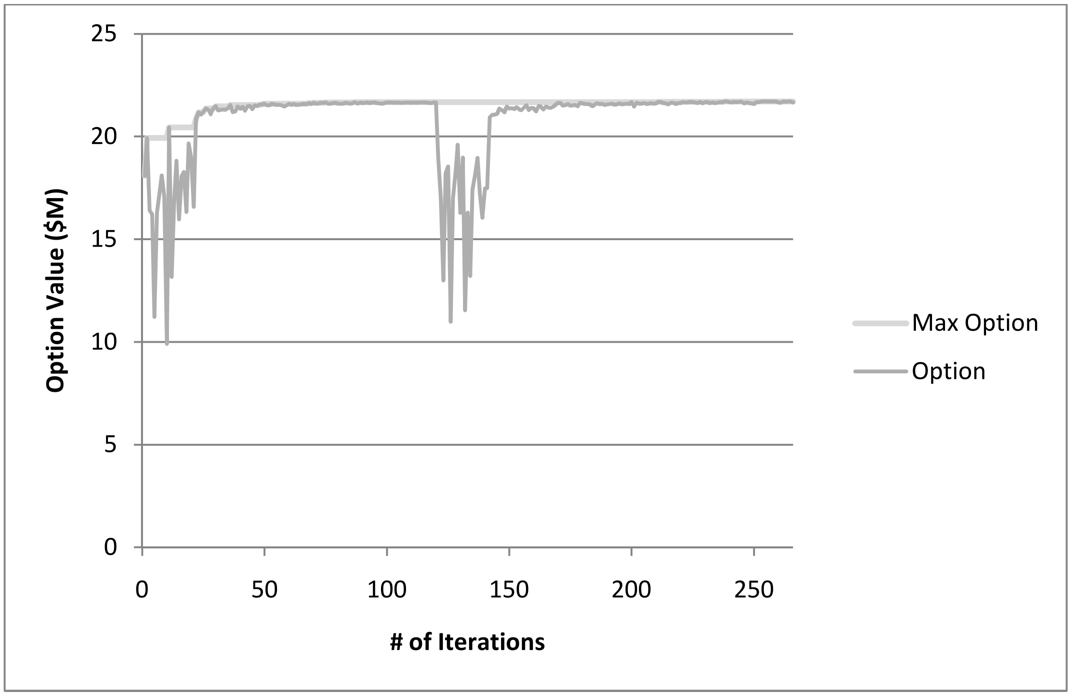

FIGURE 3-5. Option value convergence as a number of iterations of option evaluations.

The maximum option value of $21.71M is obtained in the 255th iteration, which is the 135th iteration of the 2nd local start. Contrary to the IOA solution value of TSTT = 75.942, the link designs for this solution value result in a TSTT = 76.062. This value converts to a static NPV = $19.61M, which is less than the static NPV of $19.73M from the NIDO model. However, by deferring the fixed design the option value of $21.71M is higher than the $21.50M from the fixed design option with the IOA network design.

A comparison of the NIDO, NODP, and OLIDOS results for Sioux Falls is provided in Section 3.4.

3.3.3. Ordered Link Investment Deferral Option Set

The NODP solution from Section 3.3.2 is considered as separate project options. Instead of treating each of the 10 link investments as a separate option, the pairs of opposite direction links are combined into a set of $|L|$ = 5 discrete projects to reduce computational cost: link 16 plus link 19, link 17 plus link 20, link 25 plus link 26, link 29 plus link 48, and link 39 plus link 74. The network is shown in Figure D-1 in Appendix D.

With five projects, there are 120 ordered sets to consider. For each one, the multi-option LSM approach is used to solve the option value. The same number of basis functions Π = 6 is considered. The results of each enumerated ordered project set are shown in FIGURE 3-6.

FIGURE 3-6. OLIDOS Value by Ordered Set.

The optimal ordered set is the 61st set: {3, 2, 5, 4, 1}, with a total option value of $78.67M. On the contrary, the minimum value option solution is $40.12M, with an ordered set of {1, 4, 5, 3, 2} and an optimal decision to invest on only the first project and to defer the rest. The results of the optimal solution are summarized in the table below.

TABLE 3-3. Summary of Results from OLIDOS for NODP Solution with 35% Volatility, T = 5 years, P = 300, Π = 6

Project Order	Links	Static NPV ($M)	Ordered Staging Premium ($M)	Deferral Premium ($M)	Option Value ($M)	Decision
3	25, 26	0.83	19.71	0.39	20.93	Defer
2	17, 20	0.86	18.71	0.14	19.71	Defer
5	39, 74	6.08	12.47	0.16	18.71	Defer
4	29, 48	4.99	6.85	0.63	12.47	Defer
1	16, 19	6.85	0.00	0.00	6.85	Forced Defer
SUM		19.61	57.74	1.32	78.67	

The decision is once again to defer all of the network design, but unlike the option value in Section 3.3.2, this solution includes the flexibility to invest and defer portions of the network design. The option value of the last project is actually derived from immediate investment, but since that project can only be invested on if the prior project is invested on, it is forced to defer. The full project order, total option value, and deferral decision are included in Appendix E.

The solution OLIDOS provides two significant results. First, the fixed ordering of link investments that provides the maximum option value is obtained using an exact method. Second, the value of each of the link or project options is determined so that decisions can be made with regards to which link to invest immediately and which to defer to maximize the option value.

3.4. DISCUSSION

The three models are summarized along with baseline link design values to compare the performance under the 0.35 volatility for a five year planning horizon. The OLIDOS model is run for both the NODP solution as well as the IOA solution to compare the results.

Table 3-4. Network-based Real Option Design Results with 0.35 Volatility, T=5, P=300, Π = 6

Link	Baseline	IOA	NIDO	Fixed Design Option	NODP	OLIDOS (IOA)	OLIDOS (NODP)
16	0	5.3321	5.3321	5.3321	4.9599	5.3321	4.9599
17	0	1.5323	1.5323	1.5323	2.4773	1.5323	2.4773
19	0	5.3648	5.3648	5.3648	5.4582	5.3648	5.4582
20	0	1.4891	1.4891	1.4891	2.2420	1.4891	2.2420
25	0	2.8003	2.8003	2.8003	2.7431	2.8003	2.7431
26	0	2.8635	2.8635	2.8635	3.3495	2.8635	3.3495
29	0	4.6452	4.6452	4.6452	4.7494	4.6452	4.7494
39	0	4.4403	4.4403	4.4403	3.8099	4.4403	3.8099
48	0	4.7000	4.7000	4.7000	4.6190	4.7000	4.6190
74	0	4.4140	4.4140	4.4140	4.0956	4.4140	4.0956
Deterministic TSTT	101.171	75.942	75.942	75.942	76.062	75.942	76.062
Static NPV ($M)	$0	$19.73	$19.73	$19.73	$19.61	$19.73	$19.61
1st Project	N/A	N/A	N/A	N/A	N/A	$20.99	$20.93
2nd Project	N/A	N/A	N/A	N/A	N/A	$19.89	$19.71
3rd Project	N/A	N/A	N/A	N/A	N/A	$18.90	$18.71
4th Project	N/A	N/A	N/A	N/A	N/A	$12.28	$12.47
5th Project	N/A	N/A	N/A	N/A	N/A	$6.71	$6.85
Option Value ($M)	N/A	N/A	$22.68	$21.50	$21.71	$78.77	$78.67
Deferral Premium	N/A	N/A	$1.77	$1.77	$2.10	$1.26	$1.32
Network Design Premium	N/A	N/A	$1.18	N/A	N/A	N/A	N/A
Ordered Staging Premium	N/A	N/A	N/A	N/A	N/A	$57.78	$57.74
Strategy	N/A	Invest	Defer and Redesign	Defer	Defer	Defer All Links	Defer All Links

The IOA column refers to the solution to the CNDP using IOA approach for Sioux Falls. The NIDO column computes the option value of the IOA design solution, assuming at each time state the IOA is applied to find the optimal design then. The NIDO option value $22.68M includes the flexibility to defer and re-design the network. The fixed design option column includes only the flexibility to defer and results in a slightly lower option value of $21.50M.

The Fixed Design Option is the same as NIDO, but constrains the design to be the initial design at all time states. NODP is an approximate optimal design solution using an RBF heuristic to maximize the option value as a function of both the design variables and the deferral decision. This option value of $21.71M is the maximum that can be obtained when the decision-maker can only defer a design under demand uncertainty.

Lastly, the OLIDOS model is run for the IOA design as well as the NODP design, re-computing the option values assuming the designs are de-coupled into five projects each. The option value of all the link projects together is $78.77M when using the IOA initial design and all five sets of links need to be deferred.

By using real option methodologies, it is possible to expand a stochastic network design problem to include flexibility to defer, flexibility to re-design the solution, or flexibility invest in each subset of links separately. It is also possible to optimize the design with a deferral decision with regards to the value of the investment opportunity.

"Human history becomes more and more a race between education and catastrophe." – H.G. Wells

CHAPTER 4 MOBILE SERVER RELOCATION FOR WILDFIRE PLANNING

The use of time series data for flexible management of transportation networks has been shown with real option methodologies for network design problems under uncertainty in Chapter 3. Characterizing time series data as non-stationary stochastic processes can also provide further insight to other network models such as facility location problems. By incorporating non-stationary stochastic processes in a facility relocation model, the optimal decision to relocate a set of servers can incorporate the hysteresis or inertial band due to volatility or asymmetric switching costs. This concept is explored in Chapter 4 using a wildfire planning setting, with generalizations for any resource allocation strategies in a network setting.

4.1 BACKGROUND

As discussed in Chapter 1, there is currently no systematic operational level model for using daily fire weather data to optimally redeploy resources. Four goals are achieved in this research. First, fire weather data are modeled as independent mean reverting processes, and results are shown for the estimation of the parameters for each of the CDF units (introduced in Chapter 1) in California for which data was available. Second, a new static resource allocation model that relies on seasonal average fire weather data is developed. Third a chance-constrained dynamic fire resource allocation model that relies on independent observations of fire weather data as first order Markov processes is developed. Note that the model is dynamic in the sense that the relocation model depends on the previous time-state and the updated fire weather data. The dynamic model differs from the static one in that it can incorporate forecasts of future fire conditions which are based on the mean reverting processes mentioned above. Finally, the performance of these two new models is compared against actual allocations using data from July through August 2006 and 2007.

The analysis shows that the dynamic resource relocation model using forecasts of future conditions can obtain more cost effective results than the methods currently applied as well as both the static location model and the dynamic relocation model with no forecasts. Explicitly incorporating forecasts of short term future demand in the relocation program can incorporate the value of flexibility in the positions of the resources so that excess relocations can be avoided.

4.2 LITERATURE REVIEW

4.2.1. Fire Weather Data

Fire weather data are important inputs into the decision-making processes of fire management authorities. For example, in Australia, the Bureau of Meteorology provides fire weather data as part of a national framework for fire protection. The senior meteorologists provide two sets of day to day operational outputs for each region, a public set as well as more private, detailed information for decision support to fire authorities (Gunasekera et al, 2006). In Canada an historical Large Fire Database has been developed which includes information on fire location, start date, final size, cause and suppression action for all fires larger than 200 hectares (2 km2 ~ 500 acres) in area for Canada for the 1959-1999 period (Stocks et al, 2003, Amiro et al, 2004).

Until the last few decades, fire authorities relied only on simple tools such as relative humidity or maximum daily temperature to describe fire weather. In the last few years, multiple models and indices were developed to suit the needs of local forecasters and fire authorities. Today, different land management agencies across the U.S. in both the public and private sector utilize fire indices which rely on some form of atmospheric input. Several popular indices include the Canadian Fire Weather Index (CFWI), the Keech-Byram Drought Index (KBDI), the Haines Index (HI), the Fosberg Fire Weather Index (FFWI), and the Davis Stability Index.

Until the last few years, however, only the HI was tested and scientifically validated by the peer-review process (Potter et al, 2003). The HI produces an integer between 2 and 6 with higher values indicating dry, unstable atmosphere conducive to large wildfires, and is used as an indicator that small fires could become large and erratic. The HI is currently accepted by many fire weather forecasters and fire managers as a useful tool for evaluating atmospheric conditions in fire fighting on any given day. It was shown that HI works well for plume-dominated fires, but not so much for wind-driven fires such as the Santa Ana winds in California (Potter and Martin, 2001).

Several studies done in the past for the state of California have relied on other indices based on the National Fire Danger Rating System (NFDRS), such as Gruelich and O'Regan (1982) and Haight and Fried (2007). Among the indices included in the NFDRS are the Energy Release Component (ERC) and the Burning Index (BI). The BI is an index used to relate the potential amount of effort needed to contain a single fire. The BI can be used as a guideline for staffing levels (ratings from 1 for lowest demand to 5 for the highest) (NFDRS, 2008).

4.2.2. Facility Location Problems

Facility location problems represent a rich and broad field with many different sub-problems applicable to a multitude of industries, both private and public. Among the literature reviews available, Owen and Daskin (1998), Drezner (2002, especially the chapter by Marianov and Serra) and Snyder (2006) each provide a very broad summary of the development of this field of problems, including dynamic models, public sector models, stochastic models, and scenario planning models. Brotcorne et al (2003) review ambulance location models specifically, comparing application success stories such as

Church and ReVelle's (1974) Maximal Cover Location Problem (MCLP) implemented in Austin, Texas, as well as Daskin's (1983) Maximal Expected Covering Location Problem (MEXCLP) implemented in Bangkok in 1987. In both cases, cost savings and average response times improved. Another excellent paper discussing emergency vehicle deployment is Goldberg (2004).

Berman and Odoni (1982) formulated a dynamic p-median relocation problem that responds to changes in the state of the network. The objective is to minimize steady-state expected service time and cost of relocations taking into account all the potential future allocation states. However, that formulation can quickly become intractable when a large number of facilities are involved. Berman and LeBlanc (1984) expanded on the dynamic model by making the travel times stochastic, and treating peak and off-peak periods as separate states. As the relocation cost increases due to changing states and travel times, the location/relocation strategy changes. With increasing relocation costs, the optimal strategy will involve less relocations.

Facility location problems have been used to address fire emergencies for several decades. According to Badri et al (1998), insurance service offices use the distance between customers and fire companies to rate the fire protection capabilities in different cities. Kolesar and Walker (1974) developed a computationally efficient heuristic algorithm to relocate firefighters based on calls received in districts within New York City. Their algorithm included an inconvenience cost associated with the relocations, and provided rules for constraints, such as considering relocations only for fires lasting longer than half an hour. Other urban fire covering models have been developed in the last few years as well.

However, an urban fire location model differs significantly from one applicable to wildland fires, where simultaneously occurring fires are less likely but a single fire may require multiple fire resources. A wildland fire location model should require the *k* closest facilities to cover a demand node. Marianov and Serra (2002) show that the *p*-median problem (PMP) can be modified by adding a constraint for a minimum demand threshold level to require a minimum number of servers to cover each node.

Co-location is locating multiple servers at a node, and it is another important feature to consider in wildland fire and other large-scale disaster resource models. Marianov and Revelle (1991) develop a facility co-location covering problem in which multiple response vehicles can be deployed to a single location. However, the purpose of co-location in the urban application is to deal with probabilities that the servers would be busy with another event. Their formulation indicates whether the k^{th} server is located at site *j* using a binary variable for each server.

4.2.2.1. Wildland Fire Resource Location and Deployment Models

According to Martell et al (1998), there remain many important challenges for fire deployment such as having regional fire duty officers decide each day how many fire fighters and transport vehicles should be deployed at initial attack bases. There is such a need because the 2-5% of fires that escape initial attack can cause a disproportionate amount of damage (MacLellan and Martell, 1996).

The problems in regional wildland fire resource location and deployment such as air tanker initial attack planning is that one air tanker may not be enough to cover a potential fire, and there is typically more than one air tanker co-located at an airbase. Static location models have been developed using MCLP to locate wildland fire resources. Dimopoulou and Giannikos (2001) conducted such a study using an MCLP formulation. Haight and Fried (2007) is a recent example of using a variant MCLP formulation with scenario planning to address uncertainty. They propose a multi objective covering model to deploy fire engines to stations that require a certain number of resources on a particular day. The model relies on the output of a fire occurrence simulator to determine the probability that a fire occurs at a node and also the distribution associated with the number of resources needed to contain the fire. Their formulation allows multiple resources to cover a node with co-location considered.

The trade-off between using a MCLP versus a PMP is that the PMP has an order of magnitude more decision variables but considers a more detailed objective value (in our case it's the travel time of air tankers instead of the sum of binary coverage variables). For example, if there are two nodes A and B that are within coverage distance and node A has demand requirement of 3 air tankers versus 1 from node B, an MCLP cannot differentiate between a solution of placing 4 air tankers on A or B (both cover the nodes). On the other hand, the PMP would prefer a solution with 3 air tankers on A and 1 air tanker on B because they have shorter deployment times. This differentiation is especially important in comparing the performance between a static model and a dynamic relocation model.

State-dependent probabilistic models, such as those shown in Gruelich and O'Regan (1982) and MacLellan and Martell (1996), consider the deployment of resources given transition probabilities from one state to another. While this approach allows multiple resources to be assigned to a demand node, it does not share the resources across nodes the way a facility location model would.

Several papers assume or acknowledge that fire weather data can be modeled as a first order Markov process (Gruelich and O'Regan, 1982, MacLellan and Martell, 1996), and Martell (1999) proves this is the case for the Fire Weather Index in Ontario. Martell concludes that if Markov models can be used to model day to day changes in fire danger rating indices, the properties of Markov chains can be exploited to improve fire management planning.

4.3 PROBLEM STATEMENT

The problem of allocating air tankers to air bases over the course of a wildfire season to minimize the cost and time of deploying such air tankers to actual fires is examined. By using the stochastic process to forecast near term future conditions, the model incorporates hysteresis in the solution so that the value of pre-positioning resources for potential future occurrences can be explicitly accounted. While the numerical examples and problem are defined for air tanker basing, it can also be generalized for any resource location and relocation modeling for regional networks with non-stationary stochastic demand nodes that may require multiple servers to cover.

4.4 MODEL FORMULATIONS

4.4.1. Demand

Let the state of the network be defined by the demand levels at each node at a discrete point in time (a single day), where these demand levels (resource allocations based on fire weather, not on actual fire occurrences) are stochastic on a day-to-day basis for a particular season, such as July through August each year.

Since the fire weather indices can be assumed to be first order Markov processes, the BI at each node is modeled as a discretized mean-reverting Ornstein-Uhlenbeck (O-U) process. Although O-U is a continuous process, it can be shown that a discrete AR(1) process can converge weakly to the O-U process (Tanaka, 1996). Note that we start with this model rather than using the AR(1) process directly to allow for the possibility of the availability of richer continuous data in the future.

If the BI follows a discrete mean-reverting O-U process, then it can be represented by Eq. (4.1) below using three parameters: mean reversion rate α_j, mean μ_j, and volatility σ_j.

$$dq_t^j = \alpha_j(\mu_j - q_t^j)dt + \sigma_j dW_t \quad (4.1)$$

Where j is the demand node, t is a time-state, dW_t is an increment in the Wiener process (also known as Brownian motion) which is assumed to be i.i.d. N(0, t), and q_t^j is the fire weather index value at time t for node j.

The BIs for all nodes are assumed to be independent of each other, although in reality there should be some relationship between the BI and actual fire occurrences for adjacent nodes depending on the distance and geography separating the nodes. For a large regional model like the state of California however, this interdependency can reasonably be relaxed.

The parameters of Eq. (4.1) can be calibrated using a standard least squares regression such as the one described in Van den Berg (2008) for each demand node based on historical data.

Although static location models do not need to make use of this characterization of fire weather indices since they can rely on seasonal averages, the proposed dynamic relocation model can incorporate this information in its chance constraint formulation to introduce anticipation into the model. A static location model is considered first, followed by modifications to that model into the proposed relocation model.

4.4.2. Static *k*-server *p*-Median Problem (KPMP)

Let a network be defined as G(N,A) of N nodes and A arcs with fixed travel times and demand for fire protection be linearly dependent upon the Burning Index (BI) value. Let there be a fixed number of fire resources, *P*.

The objective is to use a PMP model that includes the minimum demand thresholds discussed by Marianov and Serra (2002) while also including co-location. Instead of enumerating each k^{th} server with a binary variable, integers are used to represent the number of servers located at a node. Let's call this model formulation the *k*-server *p*-Median Problem (KPMP), where *p* servers are located such that the k_i closest servers are assigned to each demand node *i* and where co-location is possible.

$$\text{Min} \quad \sum_i \sum_j d_{ij} Z_{ij} \tag{4.2}$$

Subject to

$$Z_{ij} - X_j \leq 0, \forall i, j \tag{4.3}$$

$$\sum_j Z_{ij} \geq k_i, \forall i \tag{4.4}$$

$$\sum_j X_j = P \tag{4.5}$$

$$X_j \geq 0, \forall j, Z_{ij} \geq 0, \forall i, j, Z_i, X_j integers \tag{4.6}$$

Where

Z_{ij} = integer number of servers (air tankers) at node *j* covering node *i*

d_{ij} = matrix of distances from *i* to *j*

X_j = integer number of servers (air tankers) based at node *j*

k_i = minimum demand threshold for number of servers (air tankers) covering node *i*

P = the number of servers (air tankers) in the network

In this model, the demands at each node are shifted into the constraints and the decision variables are changed from binary variables to integer variables. Eq. (4.2) minimizes the distances of the closest servers in terms of deployment time. The objective will naturally force all the Z_{ij}'s to be zero unless they are the *k* closest servers. Eq. (4.3) guarantees that for a given base node, the number of servers assigned to cover any individual demand node does not exceed the number of servers available at that base node. Eq. (4.4) forces the *k* closest servers to a node *i* to be chosen, whether they are at the same node *j* or at different nodes. Eq. (4.5) and (4.6) are the facility budget constraint and non-negativity constraints. The Burning Index (BI) is assumed to be reasonably represented by the linear equation shown in Eq. (4.7):

$$k_i = \delta q_i \qquad (4.7)$$

Where q_i is the average value of the BI over a given time period (e.g. a two month season), and δ is a scaling parameter to convert the average BI to average air tanker staffing needs. For the static model the seasonal mean value of the BI's are used. Since the staffing levels are unknown or may vary depending on location, a reasonable value of δ is chosen that is high enough to differentiate changes in the BI values but low enough to avoid hitting the maximum P facilities ceiling. In addition, the relationship between BI and air tankers needs is assumed to be linear. This is backed up by the National Fire Danger Rating System provided by the National Oceanic and Atmospheric Administration (NFDRS, 2008). Naturally, if this function were better understood then additional relevant information could easily be incorporated into the model.

The calibration of δ is important for actual implementation, but for comparison between different model performances, a reasonable value should suffice. Note that a promising topic of future research would be to examine alternative methods for modeling and calibrating this demand constant δ.

There are $N(N+1)$ decision integer variables and $2N^2+2N+1$ constraints in this problem. If the k_i are rounded up to integers beforehand, the problem would have only N integers and N^2 continuous variables. The integer k_i's will cause the Z_{ij}'s to naturally converge to integer solutions unless two nodes share precisely the same distance value. Even then, the objective value would be the same for either case.

For example, consider a 3-node, serially linked network with P = 2, equal link lengths and servers that can only be located at node 1 or 3. Let's say k_2 = 2. The set of optimal solution results would be $\{Z_{21}, Z_{23}\} = \{[0,2], 2 - Z_{21}\}$, except that constraints (4.3) and (4.5) would force the feasible optimal solution to be $\{Z_{21}, Z_{23}\}=\{1,1\}$.

As this is a more complex version of the *p*-median problem, it is also NP-Complete. A branch and bound approach can be used for the simplified California network shown in the example, but for more complex problems with many additional decision variables a heuristic will be necessary.

4.4.3. Chance-constrained Dynamic *k*-server Relocation Problem (CDKRP)

Instead of a static, seasonal location model, consider a day-to-day model where the resource basing relocation decisions need to be made. The demand in the static model, k_i, is now a function of a stochastic variable q_i representing a fire weather index that changes day-to-day.

A chance-constrained dynamic relocation formulation of the KPMP is developed to minimize the deployment time for each time-state and to consider relocation if the benefit of a relocation is greater than the cost of transportation. The relocation is determined from anticipating the expected deployment times over a forecast time horizon *T* (e.g. 7-day forecast) based on the current day's fire weather data. In other words, if the fire weather station at some node updates its BI value to a significantly high value, it is more likely to be high in the next *T* days depending on its characterized mean-reversion process.

The idea of optimizing the locations based on multiple time periods is similar to one proposed by Repede and Bernardo (1994). Their model, referred to as the

TIMEXCLP approach, considers an expected covering problem for multiple time periods. The CDKRP uses a *p*-median problem where the future demand is obtained from the expected value of the demand using the mean-reverting process. The following model is proposed, which shall be called a Chance-constrained Dynamic *k*-server Relocation Problem (CDKRP):

$$\Phi_t = \min_X \sum_{\tau=t}^{t+T} \sum_i \sum_j d_{ij} Z_{ij}^{\tau} + \lambda \sum_r \sum_s c_{rs} Y_{rs} \quad (4.8)$$

Subject to

$$Z_{ij}^{\tau} - X_j^t \leq 0, \forall i, j, \tau \in [t, t+T] \quad (4.9)$$

$$\sum_j Z_{ij}^{\tau} \geq \min(P, E[k_j^{\tau}] + 1.645\delta\sigma_j\sqrt{\tau - t}, \forall i, \tau \in [t, t+T] \quad (4.10)$$

$$\sum_j X_j^t = P \quad (4.11)$$

$$-S_r - X_r^{t-1} + X_r^t \leq 0, \forall r \quad (4.12)$$

$$-D_s + X_s^{t-1} - X_s^t \leq 0, \forall s \quad (4.13)$$

$$\sum_s Y_{rs} = S_r, \forall r \quad (4.14)$$

$$\sum_r Y_{rs} = D_s, \forall s \quad (4.15)$$

$$X_j^t \geq 0, \forall j, t, Z_{ij}^{\tau}, X_j^t, Z_{ij}^{\tau}\ integers \quad (4.16)$$

$$Y_{rs} \geq 0, \forall r, s \quad (4.17)$$

Where

Φ_t is the expected present and future savings in deployment time at time-state *t*

t is a time-state from 0 up to the end of the season

T is a forecast horizon used for anticipating short term demand

Z_{ij}^{τ} is an integer number of air tankers at node *j* covering node *i* at time-state *τ* ≤ *t*+*T*

X_j^t is an integer number of air tankers based at node *j* at time-state *t*

d_{ij} = matrix of distances from *i* to *j*

k_j^{τ} = minimum demand threshold for number of servers (air tankers) covering node *i*

λ is a scalar conversion of the relocation cost to the value of improved deployment time

c_{rs} is the cost of transport from node r to node s

Y_{rs} is the flow from node r to node s if a relocation occurs

Sr is a dummy variable for the surplus air tankers

D_s is a dummy variable for the demand for air tankers

Eq. (4.8) and (4.9) are similar to Eq. (4.2) and (4.3) except they include the time dimension and incorporate the additional weighted cost of relocation as a Hitchcock Transportation Problem (Sheffi, 1985). To account for the volatility in the demand, chance constraints with 90% likelihood of meeting the stochastic demand are used. In other words, Eq. (4.10) represents the $\Pr\left(\sum_j Z_{ij}^{\tau} - k_j^{\tau} \geq 0.90\right)$ for each time-state τ. Since the demand variables are Wiener processes that follow independent normal distributions with variance $Var\left[h_j^{\tau}\right] = (\tau - t)\sigma_j^2$, Eq. (4.10) can be used to capture the chance constraints. An additional condition in Eq. (4.10) is included for situations where the chance constraint exceeds the maximum number of air tankers available so that the solution will remain feasible. Eq. (4.11) is a budget constraint.

For air tanker relocation costs, the minimum cost relocation path should be taken, subject to constraints Eq. (4.12) – (4.15). Eq. (4.12) – (4.13) assigns the differences in facility locations to supply and demand at each node using dummy variables. Eq. (4.14) – (4.15) are flow conservation constraints. The remaining constraints are non-negativity and integer constraints. The need for relocation will be determined if there is a difference between X_j^{t-1} and X_j^t for any node *j*. This leads to a tradeoff in the objective function between minimizing deployment time to cover fire outbreaks versus minimizing the costs of relocation.

Although no monetary value is used for decision-making here, a conversion rate of λ = 1 may not be appropriate because one unit is in terms of air tanker transport cost while the other is the risk of loss due to the time it takes for air tankers to make an initial attack on a fire. Many other costs are not directly accounted for though careful selection of the value of λ could in fact account for these. In the numerical test, several λ values are compared with July to August 2006 and 2007 observed fire weather data and observed fire occurrences.

As a relocation problem with chance constraints, this problem can still be solved with the same branch and bound algorithm as for KPMP. The problem needs to be solved once for each day of the season instead of just once for the whole season, and then daily averages can be computed to compare the objective values. The number of Z_{ij} variables is increased by *T*-*t* times, but the number of X_i variables remains the same. If the k_i are rounded up to integers, the problem remains a mixed integer program with *N* integer decision variables and $N^{2(T-t+1)}+2N$ continuous variables that would naturally converge to integer values.

Note that compared to Berman and Odoni's (1982) work, the use of chance constraints with independent demand nodes allow the problem to remain tractable for the network and number of facilities used in the numerical test in Section 4.5. Nevertheless, the mixed integer program becomes immense if long-term forecast such as a 2-month horizon is considered (and may be unrealistic in any case – it's the same reason weather forecasters do not provide 2-month weather forecasts) so smaller increments are considered, such as 3-day and 7-day forecasts.

FIGURE 4-1 presents a conceptual representation of how the models respond (or in the case of static location model, don't respond) to changes in fire prediction data day by day. The forecasting feature of the dynamic relocation models with 3- and 7-day horizons is depicted in the bottom tab as a trajectory of the stochastic weather data. By incorporating this forecasting, there's anticipation built into the model, which in turn reduces costs from excess relocations. This conclusion can be drawn from the following numerical test.

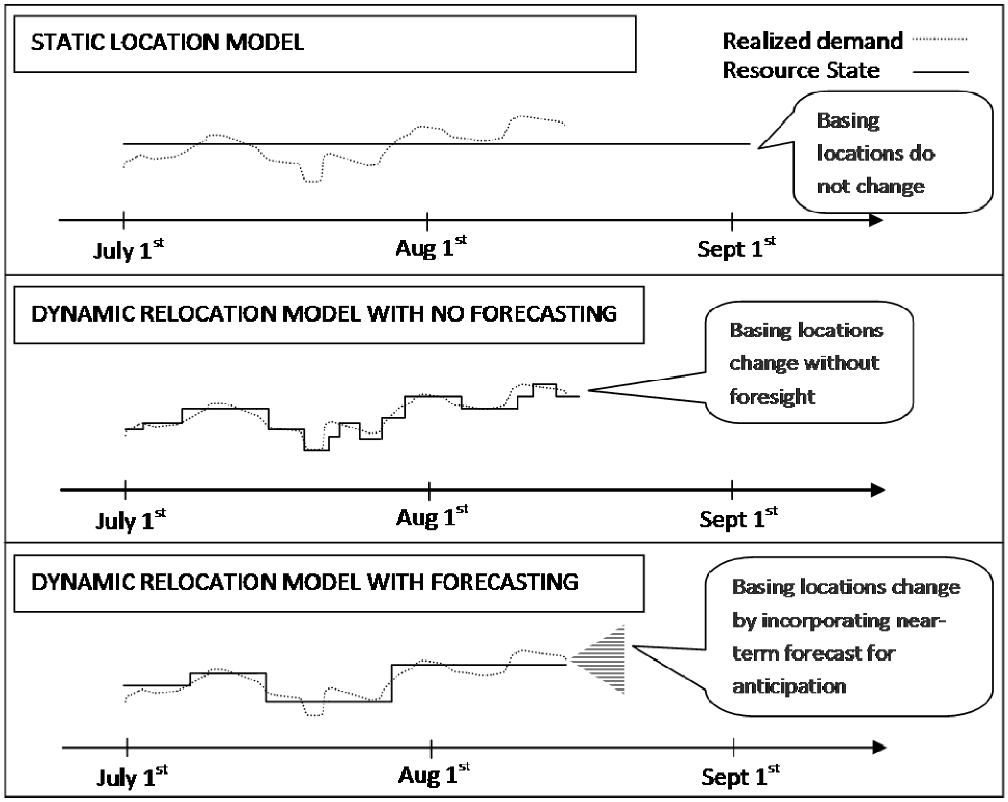

FIGURE 4-1. A conceptual illustration of the three models

4.5 NUMERICAL TESTS

Air tanker initial attack deployment is chosen as an example of California wildland fire operation to analyze because the air tankers can fly from any one node to another, so the allocation problem will be kept simplified to a transportation problem of *N* nodes to *N* nodes. Additionally, air tankers can feasibly aid other regions in the state so a regional scenario with large scale fire occurrences can be tested against.

For this numerical test, the six contract counties are omitted and only the counties with air bases are included in the network. Since no data could be found for fire weather data stations at SCU and NEU, those two CDF's are also omitted. The remaining 12 CDF's are shown below. The acronyms FFS and AAB refer to forest fire stations and air attack bases, respectively. The fire weather data is obtained from the FFS locations and the travel distances are based on the AAB locations. Details of the distances are provided in Appendix F.

1. BEU – San Benito-Monterey (FFS @ Hollister, AAB @ Hollister)
2. BTU – Butte (FFS @ Cohasset, AAB @ Chico)
3. FKU – Fresno-Kings (FFS @ Hurley, AAB @ Fresno)
4. HUU – Humboldt-Del Norte (FFS @ Alder Point, AAB @ Rohnerville)
5. LNU – Sonoma-Lake-Napa (FFS @ Santa Rosa, AAB @ Sonoma)
6. MEU – Mendocino (FFS @ Laytonville, AAB @ Ukiah)
7. MVU – San Diego (FFS @ Julian, AAB @ Ramona)
8. RRU – Riverside (FFS @ Beaumont, AAB @ Hemet)
9. SHU – Shasta-Trinity (FFS @ Redding, AAB @ Redding)
10. SLU – San Luis Obispo (FFS @ La Panza, AAB @ Paso Robles)
11. TCU – Tuolumne-Calaveras (FFS @ Green Spring, AAB @ Columbia)
12. TUU – Tulare (FFS @ Milo, AAB @ Porterville)

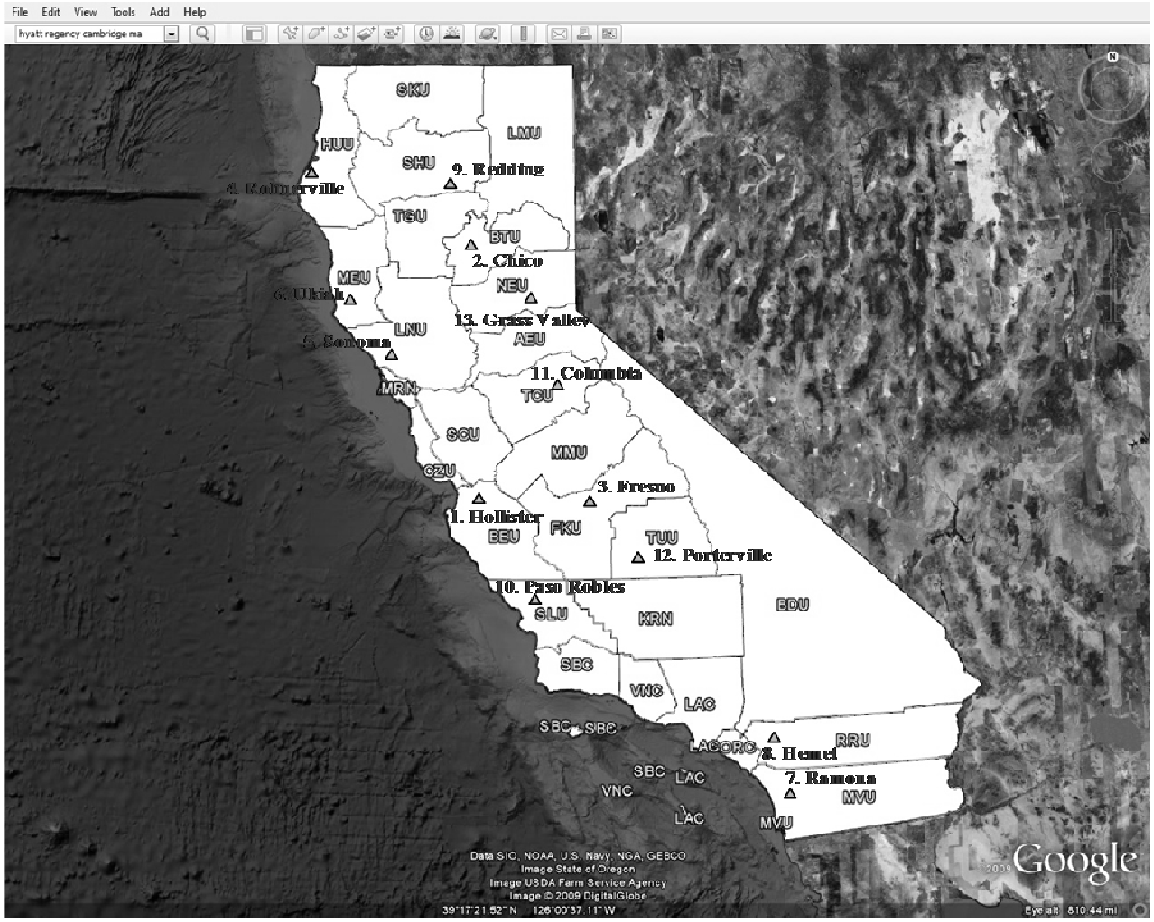

FIGURE 4-2. CDF Air Bases in California (GIS data from http://wildfire.cr.usgs.gov/fireplanning)

The locations of these air bases are shown in FIGURE 4-2. Because not all the air base CDF's are being included in the test network, not all the air tankers will be used. Out of the 23 air tankers contracted among the CDF's, 20 of them are located in the air bases being analyzed in the test network.

The CDF owns and operates S-2T air tankers, which can each carry up to 1200 gallons (4542 L) of retardant and have maximum operating speeds of 270 mph (435 kph). For the purposes of this experiment, the assumed average speed is 250 mph (402 kph). These values are used to calculate the d_{ij} values in the CDKRP, which are travel times in units of hours.

A demand parameter δ = 1/7 is chosen empirically as the linear factor to convert the BI to air tanker demand, although in a real application it should be calibrated to match the CDFs' average air tanker response needs to the staffing levels from the NFDRS. Using the demand parameter, a mean BI of 30 would translate to a 4-5 air tanker demand threshold for the KPMP.

TABLE 4-1. California Test Network with Least-Squares Estimated Parameters

CDF	Unit Name	AAB	FFS	Mean BI	Reversion Rate	Volatility	R^2
BEU	San Benito-Monterey	Hollister	Hollister	29.09	0.4427	8.5231	0.39
BTU	Butte	Chico	Cohasset RAWS	72.91	0.3056	25.8443	0.55
FKU	Fresno-Kings	Fresno	Hurley RAWS	68.26	0.1567	15.7480	0.74
HUU	Humboldt-Del Norte	Rohnerville	Alder Point	39.09	0.2568	5.3092	0.61
LNU	Sonoma-Lake-Napa	Sonoma	Santa Rosa	33.58	0.6400	11.8233	0.29
MEU	Mendocino	Ukiah	Laytonville	43.39	0.2143	4.8636	0.66
MVU	San Diego	Ramona	Julian	69.49	0.1411	26.0423	0.76
RRU	Riverside	Hemet	Beaumont	35.82	0.4806	12.7786	0.42
SHU	Shasta-Trinity	Redding	Redding	39.82	1.2090	16.3320	0.09
SLU	San Luis Obispo	Paso Robles	La Panza	38.29	0.7480	13.3935	0.22
TCU	Tuolumne-Calaveras	Columbia	Green Spring	36.28	0.9380	11.7410	0.15
TUU	Tulare	Porterville	Milo	28.42	0.2204	5.4760	0.72

TABLE 4-1 shows the network of air tanker bases in more detail. BI data was collected for each FFS from July through September, from 2001 through 2006 from Fire and Weather Data made available by the U.S. national wildfire coordinating group (http://www.nwcg.gov/) and using the FireFamily Plus software. A sample output is shown in Appendix F. The reversion rate α and volatility σ for each demand node were estimated. The R^2 values vary quite significantly by location, which suggests there are many other node-specific factors that can come into play in fitting the BI data to a mean-reverting process.

A major assumption is made that the FFS weather stations chosen for each of the CDF units are representative of the state of fire weather for the whole unit. While this assumption may be invalid for especially large regions, the same dataset is used for both models so that initial comparisons between a static model and a dynamic model can be made. Further research should look at multiple FFS weather station data sources to represent CDF units.

4.5.1. Static Location Model

Using the mean BI data, a static location basing plan was developed for the period of July 1st through August 31st for 2006.

In TABLE 4-2, the actual numbers of air tankers currently allocated to those 12 nodes are shown on the second right-most column, obtained from CDF (2008). The KPMP solution has a total deployment time (objective value) of 31.46 air tanker-hours needed to meet the average seasonal demand constraints, whereas the actual locations require 34.17 air tanker-hours. The difference in expected deployment time is 8%.

TABLE 4-2. KPMP Solution, 2006

Node #	CDF	Demand Constraint	KPMP No. Air Tankers	KPMP Deployment Times	Actual No. Air Tankers	Actual Deployment Times
1	BEU	4.16	1	1.66	2	1.44
2	BTU	10.4	2	5.44	1	4.58
3	FKU	9.8	3	4.45	1	4.31
4	HUU	5.58	0	2.80	1	2.41
5	LNU	4.8	1	0.95	2	1.10
6	MEU	6.2	4	1.48	2	1.92
7	MVU	9.9	0	5.52	2	8.34
8	RRU	5.12	6	1.50	2	2.70
9	SHU	5.69	0	2.16	2	1.78
10	SLU	5.47	0	2.16	2	1.80
11	TCU	5.18	1	2.32	2	2.12
12	TUU	4.06	2	1.02	1	1.67
	TOTAL		20	31.46	20	34.17

4.5.2. Dynamic Relocation Model

Initial results using different conversion rates between relocation cost to deployment cost, λ = 0.10, 1.0, and 10.0, were obtained for the relocation model based on observed fire weather for the months of July and August in 2006 and 2007. Two different forecast horizons were used, a 3-day forecast and a 7-day forecast. In addition, a 0-day forecast is obtained to represent a No Forecast model that uses only the current day's values. The results are shown in TABLE 4-3 and TABLE 4-4 for 2006 and 2007, respectively.

TABLE 4-3. Dynamic Relocation Results for July-Aug 2006

NO FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	57.687	50.295	58	6.315
1	57.687	51.435	42	24.516
10	57.687	57.687	0	0
3-DAY FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	57.687	55.434	55	3.597
1	57.687	56.261	33	13.535
10	57.687	55.926	4	17.300
7-DAY FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	57.687	56.653	32	1.802
1	57.687	57.299	9	5.446
10	57.687	57.115	3	21.000

TABLE 4-4. Dynamic Relocation Results for July-Aug 2007

NO FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	101.582	90.908	62	7.916
1	101.582	93.079	48	30.992
10	101.582	101.582	0	0
3-DAY FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	101.582	83.613	56	4.133
1	101.582	82.882	37	16.929
10	101.582	95.245	3	13.900
7-DAY FORECAST				
λ	KPMP Avg Daily Deployment Time	CDKRP Avg Daily Deployment Time	CDKRP No. of Times One or More Relocations Occur	CDKRP Seasonal Relocation Costs
0.1	101.582	90.119	47	2.658
1	101.582	86.987	28	11.489
10	101.582	89.863	1	21.000

The "Avg Daily Deployment Time" columns represent the expected deployment times each day for the minimum demand thresholds defined by the observed fire weather. Note that the KPMP Avg Daily Deployment Times in 2006 are in actuality much higher than what is computed for the seasonal average of 31.46 from TABLE 4-2. This is the result of the fluctuations in the observed fire weather during the 62-day season. Also note that in 2007 the general fire weather was more severe, leading to higher deployment times.

For example, with respect to the "CDKRP No. of Times One or More Relocations Occur", there were 58 instances when one or more relocations were required in the two month period in 2006 for λ = 0.1 based on the top row of TABLE 4-3.

The "CDKRP Seasonal Relocation Costs" represent the total costs accrued in the two months due to all the relocations, adjusted with the λ value to be in units of deployment time. For λ = 10, sometimes the transport cost of the CDKRP is so high relative to the improvement in expected deployment time that no relocations are made, resulting in the same solution as KPMP.

In the following section, the models for λ = 0.1 and 1 are validated against existing and static location models using actual fire occurrences in 2006 and 2007. The λ = 10 results are omitted since they are so similar to the static location model results.

4.5.3. Validation of Models with Actual Fires

To validate the performance of the static location model compared to the dynamic relocation model with and without forecasting, actual fire occurrence data from July through August in 2006 and 2007 are used. The California Fire Alliance (CFA) (2008) provides reported fire occurrence data by CDF unit and by magnitude measures such as number of GIS acres.

To compare the performance of the models in with actual fire occurrences, the *k* closest air tankers determined from the models are assigned to each fire occurrence. The actual deployment times are summed up and weighted by the distribution of the GIS acres for the season to account for the severity of the fires. The total relocation costs are divided by the number of fires in that season and added to the acre-weighted deployment times to obtain what we call the “Net k-Deployment Time”. TABLE 4-5 shows the validation results for 2006 and TABLE 4-6 shows the results for 2007, assuming each fire requires all 20 air tankers in the network, weighted by GIS acre percentage. The results of the CDKRP for no forecast, 3-Day and 7-Day forecast are compared to the KPMP solution and the actual base assignments.

TABLE 4-5. Actual Fires and Acre-Weighted Deployment Time from ALL Air tankers, 2006

					KPMP	CDKRP, No Forecast		CDKRP, 3-Day Forecast		CDKRP, 7-Day Forecast	
Fires:	**CDF:**	**CDF#:**	**GIS Acres**	**Actual**	**Static**	**λ=0.1**	**λ=1.0**	**λ=0.1**	**λ=1.0**	**λ=0.1**	**λ=1.0**
7/3/06	TCU	11	1997	1.031	1.082	1.183	1.172	0.857	0.857	0.691	0.710
7/4/06	TCU	11	150	0.077	0.081	0.097	0.096	0.067	0.068	0.052	0.053
7/5/06	BEU	1	17	0.009	0.009	0.010	0.010	0.008	0.008	0.006	0.006
7/15/06	BEU	1	249	0.127	0.133	0.134	0.140	0.102	0.107	0.094	0.090
7/15/06	MVU	7	290	0.290	0.246	0.235	0.219	0.297	0.292	0.308	0.308
7/20/06	MVU	7	317	0.318	0.269	0.253	0.257	0.316	0.316	0.337	0.338
7/20/06	TUU	12	1360	0.807	0.716	0.713	0.727	0.636	0.636	0.589	0.593
7/22/06	BEU	1	14509	7.410	7.776	6.946	7.697	5.859	5.948	5.493	5.375
7/22/06	FKU	3	9435	4.983	4.719	4.083	4.597	3.595	3.527	3.245	3.283
7/23/06	BEU	1	86	0.044	0.046	0.042	0.043	0.033	0.034	0.032	0.032
7/23/06	FKU	3	229	0.121	0.115	0.102	0.110	0.085	0.085	0.081	0.080
7/27/06	TCU	11	200	0.103	0.108	0.096	0.100	0.074	0.073	0.069	0.068
7/29/06	BEU	1	247	0.126	0.132	0.127	0.120	0.104	0.097	0.095	0.093
8/1/06	BEU	1	11	0.006	0.006	0.006	0.006	0.005	0.005	0.004	0.004
8/2/06	TCU	11	87	0.045	0.047	0.043	0.044	0.034	0.033	0.030	0.029
8/6/06	MVU	7	9	0.009	0.008	0.008	0.008	0.009	0.009	0.010	0.010
8/8/06	RRU	8	126	0.115	0.098	0.095	0.096	0.110	0.110	0.120	0.120
8/11/06	MVU	7	43	0.043	0.036	0.041	0.044	0.046	0.047	0.046	0.046
8/26/06	MVU	7	8	0.008	0.007	0.007	0.007	0.008	0.008	0.008	0.009
Weighted Avg Deployment Time (hrs)				15.67	15.64	14.22	15.50	12.25	12.26	11.31	11.25
Average Relocation Costs/Fire				0.0	0.0	0.33	1.29	0.19	0.71	0.09	0.29
Net k-Deployment Time (k = 20)				15.67	15.64	14.55	16.79	12.44	12.97	11.40	11.54
% Deployment Time Reduction				0%	0.2%	7.1%	-7.1%	20.6%	17.2%	27.2%	26.4%

TABLE 4-6. Actual Fires and Acre-Weighted Deployment Time from ALL Air tankers, 2007

					KPMP	CDKRP, No Forecast		CDKRP, 3-Day Forecast		CDKRP, 7-Day Forecast	
Fires:	**CDF:**	**CDF#:**	**GIS Acres**	**Actual**	**Static**	**λ=0.1**	**λ=1.0**	**λ=0.1**	**λ=1.0**	**λ=0.1**	**λ=1.0**
7/1/07	MVU	7	95	0.396	0.335	0.328	0.335	0.406	0.412	0.409	0.411
7/1/07	BEU	1	49	0.104	0.109	0.101	0.100	0.077	0.074	0.076	0.074
7/2/07	TCU	11	101	0.217	0.228	0.210	0.220	0.149	0.149	0.146	0.143
7/6/07	MVU	7	136	0.567	0.480	0.465	0.462	0.573	0.584	0.593	0.553
7/29/07	RRU	8	224	0.850	0.728	0.654	0.653	0.823	0.823	0.870	0.856
8/1/07	SLU	10	29	0.070	0.068	0.064	0.064	0.056	0.056	0.056	0.056
8/10/07	FKU	3	5671	12.462	11.803	10.590	10.244	7.987	8.188	7.914	7.818
8/18/07	FKU	3	92	0.202	0.191	0.175	0.163	0.130	0.131	0.130	0.131
8/19/07	SLU	10	17	0.041	0.040	0.039	0.039	0.032	0.032	0.031	0.031
8/23/07	SLU	10	13	0.031	0.030	0.028	0.027	0.026	0.025	0.025	0.025
8/24/07	BEU	1	19	0.040	0.042	0.040	0.038	0.030	0.030	0.030	0.029
8/28/07	FKU	3	151	0.332	0.314	0.283	0.281	0.214	0.222	0.216	0.217
8/30/07	SLU	10	20	0.048	0.047	0.053	0.053	0.041	0.041	0.040	0.040
8/30/07	BEU	1	422	0.897	0.941	0.860	0.851	0.677	0.689	0.646	0.667
8/30/07	BEU	1	19	0.040	0.042	0.039	0.038	0.030	0.031	0.029	0.030
Weighted Avg Deployment Time (hrs)				16.30	15.40	13.93	13.57	11.25	11.49	11.21	11.08
Average Relocation Costs/Fire				0.0	0.0	0.53	2.07	0.28	1.13	0.18	0.77
Net k-Deployment Time (k = 20)				16.30	15.40	14.46	15.64	11.53	12.62	11.39	11.85
% Deployment Time Reduction				0%	5.5%	11.3%	4.0%	29.3%	22.6%	30.1%	27.3%

TABLE 4-5 and TABLE 4-6 show the computations for obtaining the acre-weighted air tanker-deployment times plus the average relocation costs (Net 20-Deployment Time). However, that's assuming that an average actual fire outbreak requires 20 air tankers to be deployed. Since that is not actually known, a sensitivity analysis is conducted using the same calculations for every other number of air tankers. The results of the validation for every *k* value (this implies that the k-closest air tankers out of the 20 are assigned) for both 2006 and 2007 are shown in FIGURE 4-3 and FIGURE 4-4, respectively.

FIGURE 4-3 indicates that if the average actual wildland fire outbreak requires only a few air tankers, then static location model and actual locations are sufficient. This makes intuitive sense because relocation costs would overwhelm the relocation models – particularly for this extreme case. Note that the static model is approximately the same as the actual locations, and is slightly worse for most cases shown for the range of 1 – 20 air tankers.

As fires become more severe and require on average more than 7 air tankers, the relocation models with forecasting begin to dominate in performance. The benefit of having forecasting is clear in this figure as well – the relocation model without forecasting has no foresight and is much less cost effective. In fact, when λ exceeds a certain point it actually becomes less cost-effective than the static model using seasonal average fire weather data and actual locations for all severity scenarios.

The difference between the No Forecast relocation model and the 3-day or 7-day forecast models is the value of incorporating hysteresis in the relocation.

The figure also shows that generally the 7-day forecasts perform best in the mid- to high- severity ranges of fire outbreaks requiring many air tankers.

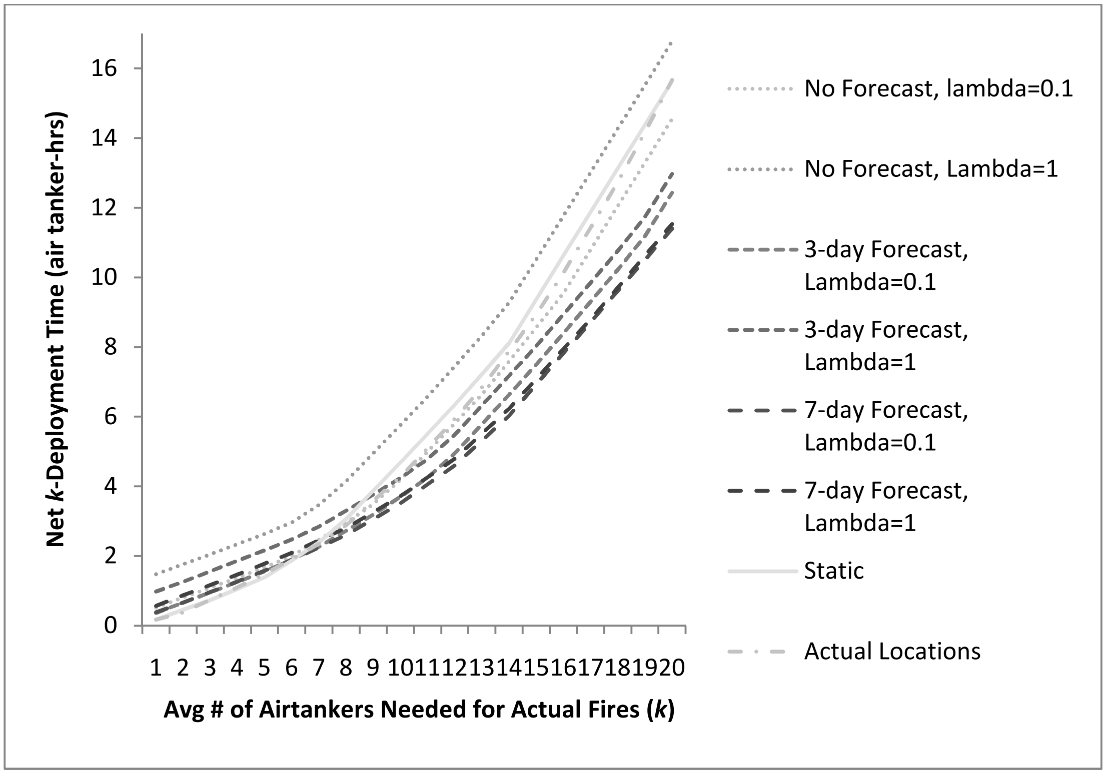

FIGURE 4-3. 2006 Sensitivity Analysis of Acre-Weighted Deployment Air Tanker-Hours versus Average Number of Air Tankers Needed for Actual Fires

The 2007 data shown in FIGURE 4-4 looks a little different (there's a sharper divergence between models). Again, the no forecast relocation model can be less cost-effective than static model. The relocation models with forecasting surpass the static models for actual fires requiring on average 7 or more air tankers.

The 2006 and 2007 data shows that relocation models using fire weather data without any foresight can result in poor performance regardless of the *k* value for actual fires. On the other hand, including a forecast horizon in the model can potentially reduce deployment times of required air tankers by 20 – 30% for the most severe wildland fire scenarios, and be approximately equivalent or slightly worse for seasons with small wildland fire outbreaks. As average fires become more severe in the future in the face of global warming and worsening environmental conditions, these types of models will be necessary to reduce the risk of loss from major fire outbreaks.

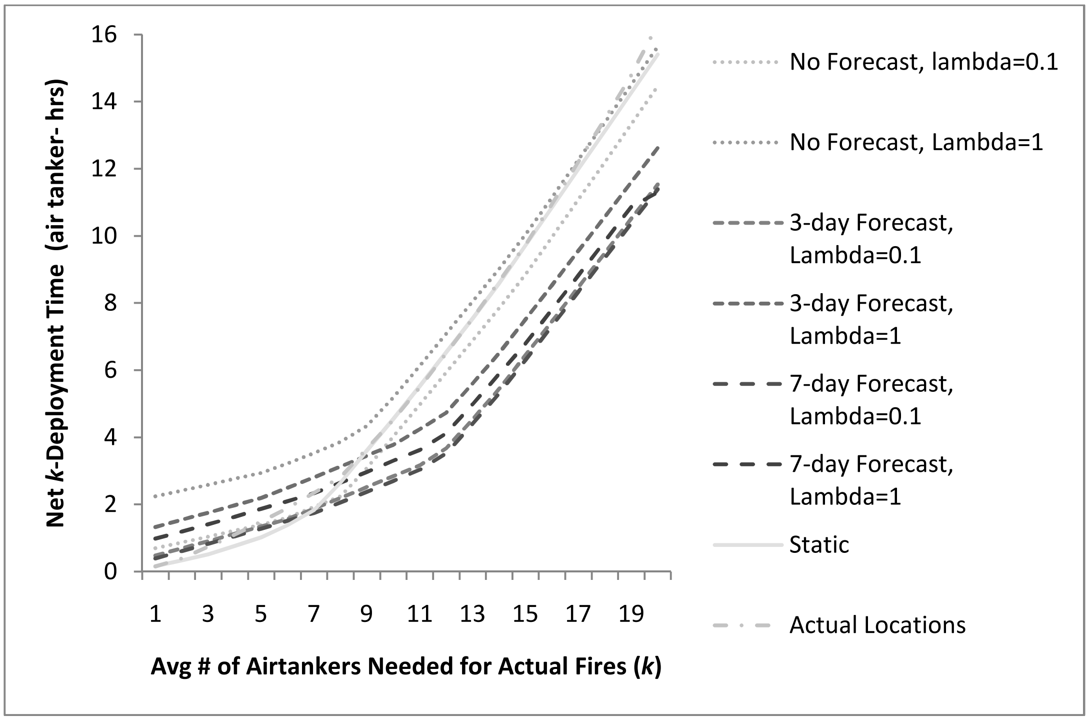

FIGURE 4-4. 2007 Sensitivity Analysis of Acre-Weighted Deployment Air Tanker-Hours versus Average Number of Air Tankers Needed for Actual Fires

Two issues should be kept in mind with the actual fire occurrence performance measures; the first is the accuracy of the BI as a fire prediction index. The dynamic relocations are based on the assumption that the BI strongly correlates to the occurrence of a magnitude-weighted fire occurrence. If that is not the case, the dynamic models may perform even better using a more accurate fire weather index. The second issue is the use of select weather stations as "representative" stations for each CDF unit. A more robust model in the future should include data from multiple weather stations in each unit.

Note that the proposed models do not explicitly consider the possibility that a server will be simultaneously deployed to two nodes. The topology of the example network (sparse) makes such an assignment relatively unlikely. However if there was a need to extend this to an urban problem or even one in which simultaneous and proximate fires might occur then similar results can be obtained by increasing the demand parameters used in the model to account for a chance of being busy.

The main conclusion would remain – when fires are less severe a static model outperforms a dynamic one (the basing decisions should be very effective). However when fires are more severe, and therefore relocations will help agencies respond with a large number of servers quickly, dynamic basing (taking into account fire weather data) outperforms static basing. In the state of California, fires are increasing in size and intensity – these claim more lives and acres each year. Therefore, new methods to most effectively use resources are essential.

4.6 DISCUSSION

The proposed models present possible new statewide regional strategies for the ever-growing problem of wildland fire planning by integrating data from ever-improving fire weather indices and facility location models. Earlier work in identifying fire weather indices as Markov processes was extended to show how fire weather data could be modeled as mean-reverting stochastic processes. Two multiple-server models were developed to meet the needs of the unique conditions of wildland fire planning: large-magnitude fires requiring regional coordination in initial attack from air bases with multiple air tankers. The results show promise for dynamic relocation models with forecasting and argue against the use of static (seasonal) location models for regions with typically severe wildland fire outbreaks. For any setting, dynamic relocation without forecasting can be less cost-effective than the static model.

The results here reveal new insights for state agencies; in the face of global warming and ever-more severe wildland fires, it is crucial to identify the threshold *k* value for which it would be more cost-effective to adopt a centralized dynamic relocation model using fire weather data and forecast horizons.

The results can also be generalized for resource allocation in a network. Incorporating the parameters of the stochastic process as chance constraints adds value of flexibility to an existing position or solution. This concept can have tremendous benefit in humanitarian logistics problems involving large scale consequences where pre-positioning with adaptive intelligence can save decision-makers the cost of unnecessary switches.

"Each problem that I solved became a rule which served afterwards to solve other problems." – Rene Descartes

CHAPTER 5 FASTER CONVERGING HEURISTICS FOR CONTINUOUS NETWORK DESIGN-BASED MODELS

As discussed in Chapter 2, the state of the art in network design models has broadened significantly in scope and applicability to meet the needs of increasingly complex transportation network management problems. Whereas early development in network design models focused primarily on solution algorithms, the current breadth of developments cover a wide range from different network design problem formulations such as capacity expansions (Yang Bell, 1998), network toll pricing (Yang and Bell, 1997), or signal timing (Ceylan and Bell, 2004), to different objectives such as reliability (Chen et al, 2002), social and spatial equity (Yang and Zhang, 2002), incorporating flexibility with deferral options as discussed in Chapter 3, and complex solution algorithms such as stochastic programming (Chen and Yang, 2004) and robust optimization (Karoonsoontawong and Waller, 2007), to name a few.

While some of the models feature multi-objective or mixed integer formulations, the network models mentioned above share an underlying bi-level optimization program with continuous decision variables representing a managing agent's decision-making at the upper level and the response of individual users at the lower level. As pointed out by Yang and Bell (1998), continuous network design problems (CNDP) are non-convex and non-differentiable, so solutions require approximate heuristics, such as sensitivity analysis based (SAB) algorithms (Yang and Bell, 1998, Yang and Bell, 1997), other derivative-based methods (Gao et al, 2007), simulated annealing (Friesz et al, 1992) or genetic algorithms (GA) (Chen and Yang, 2004).

GA methods are popular with researchers because of their versatility in handling a vast array of different non-convex functions without the need to differentiate the functions, for both continuous and discrete decision variables. GA methods are evolutionary methods that rely on stochastic convergence of the optimum over many iterations. Since each iteration of GA involves evaluation of a lower level network equilibrium, this method can have very slow convergence rates for large scale networks. This limitation makes it difficult to apply many of these network design models to practice, especially for more complex hierarchical models such as multi-objective network design problems or the network-based real option models in Chapter 3.

This chapter is divided into two main parts. The first (Sections 5.1 – 5.3) considers the multi-start MSRBF algorithm developed by Regis and Shoemaker (2007) as a faster alternative to other global heuristics for solving continuous network design problems. The second (Sections 5.4 – 5.5) proposes a multi-objective solution algorithm based on the MSRBF algorithm, which is then applied to the robust toll pricing problem in Chapter 6.

5.1 BACKGROUND

An alternative method is considered and proposed as a faster global heuristic for the CNDP. This method is based on the Metric Stochastic Response Surface (MSRS) method developed by Regis and Shoemaker (2007). MSRS is a global stochastic optimization approach that can use radial basis functions (RBF's) to intelligently guess the next point to evaluate using interpolation (MSRBF). The multi-start local MSRBF algorithm is shown to work extremely well for continuous, high-dimensional, multimodal, computationally expensive functions with box constraints. Most importantly, it has been shown to converge faster than other non-derivative based heuristics such as GA and SA, for functions with up to 14 dimensional variables.

The introduction and evolution of the method is described next, followed by a proposed design of the algorithm for the CNDP. A numerical test is performed to compare the convergence rate in terms of the number of user equilibrium (UE) evaluations of the proposed method with a genetic algorithm based on the parameters set in Chen and Yang (2004). This comparison is conducted for the classical Sioux Falls

South Dakota network for both the given origin-destination (OD) flow volumes and more congested conditions. Lastly, the algorithm is applied to the larger Anaheim California network to illustrate its performance there. This would be the first test of the multi-start MSRBF algorithm on a 31-dimensional network design problem. While this research is applied specifically to the CNDP, it should be applicable to other network models such as the network toll pricing problem, as long as the decision variables are continuous variables.

5.1.1. Artificial Neural Networks

Artificial neural network algorithms have been explored as a way of avoiding computationally expensive functions by using pattern recognition. As described by Haykin (1994), neural networks are essentially nonlinear data modeling tools that utilize multiple layers of linear vectors that take input vectors and convert them to output vectors. By designing the number of layers (intermediate layers not directly interacting with the final input or output vectors are called hidden layers), the input-output mechanism, and the size of the transformation vectors, a "neural network" that is analogous to the pattern recognition abilities of the human brain can be replicated.

Xiong and Schneider (1992) used a GA with a neural network to solve a discrete network design problem. The neural network algorithm is trained to obtain user equilibrium total system travel times based on different discrete link investment allocations using a subset of discrete allocation solutions obtained from the iterative Frank-Wolfe algorithm. The trained neural network is then able to approximate solutions from new inputs much faster than the Frank-Wolfe algorithm. For a consideration of 20 investment projects, the authors showed that a 100-solution training set was sufficient for good approximation.

Wei and Schonfeld (1993) used neural networks directly to obtain estimates of total travel times for discrete network design problems with project staging. As a more complex problem with an additional time dimension, it turns out the algorithm is fairly computationally expensive even for 3 projects with 27 units in the single hidden layer.

5.1.2. Radial Basis Functions

One type of an artificial neural network used to solve a curve-fitting problem in high-dimensional space is the RBF's (Haykin, 1994). RBF's are interpolation functions composed of a linear combination of independent functions. Given a set of predetermined points in a global search, the values can be used to estimate a set of independent spanning basis functions. These RBF's can then be used to evaluate candidate points with minimal computational cost relative to that of evaluating the objective function.

These response surface optimization methods using RBF's have been developed by Gutmann (2001), with Matlab implementation by Björkman and Holmström (2000). The interpolating RBF's described by Gutmann take the form of Eq. (5.1).

$$s(x) = \sum_{i=1}^{n} \lambda_i \phi(\|x - x_i\|) + p(x), x \in \mathbb{R}^{\kappa} \quad (5.1)$$

Where $p(x)$ is a polynomial of degree m, and ϕ is suggested to be one of the following: linear, cubic, thin plate spline, multiquadrics, or Gaussian. However, results by Gutmann have shown that the multiquadrics and Gaussian functional forms do not perform as well as cubic or thin plate spline.

Regis and Shoemaker (2005) extended the method to handle constrained global optimization problems and later refined their method into two stochastic search algorithms: a global MSRBF method that alternates between varying degrees of global and local search, and a multi-start local MSRBF that searches the neighborhood of the best found solution (Regis and Shoemaker, 2007). They show that their MSRBF methods converge stochastically and that they can converge faster than some traditional methods such as simulated annealing and genetic algorithm for constrained problems, depending on the dimension of the problem. Their conclusion is that the multi-start local MSRBF performs much better than the global MSRBF for higher dimensional problems, which they tested with 14-dimensional functions.

In the following section, both the global MSRBF and multi-start local MSRBF methods are customized for the CNDP so that they can be generally applied to any network design problem with continuous decision variables.

5.2 MSRBF ALGORITHM FOR CNDP

5.2.1. Continuous Network Design Problem

The CNDP as described in Section 3.1.1.4 is generally defined as an allocation of a fixed budget to improve some continuous elements of existing links such that the overall cost flows are minimized, subject to selfish user route choice behavior and link congestion. The model formulation is shown in Eq. (3.3) – (3.9).

5.2.2. Multi-start Local MSRBF Algorithm for CNDP (RBFCNDL)

The algorithm presented here is based on the multi-start local MSRBF framework from Regis and Shoemaker (2007) and customized for the CNDP, an RBF based continuous network design algorithm (RBFCNDL). Steps 0 – 11 with accompanying descriptions are presented below with the highlights shown in FIGURE 5-1. Since the global MSRBF method (which will be labeled RBFCNDG) is similar except for a few key steps, comments are included in those sections to denote the differences where applicable.

Step 0. For RBFCNDL, Steps 1 - 11 will keep repeating until $n = N_{max}$; until then the algorithm would reset with a new initial design if $\rho = \rho_{max}$, where $\rho \equiv 0$ at start and increment up when $\zeta = \zeta_{max}$.

a. $\zeta \equiv 0$ at start and increments up when no improvement is made on the best solution found in an iteration.

b. For the CNDP an ζ_{max} = max(5, κ) and ρ_{max} = 5 are used.

Step 1. Generate an initial set of points, $\{\theta_1, \ldots, \theta_{n_0}\}$ using Latin hypercube sampling (LHS).

a. n_0 is set to 2κ, where κ is the size of $\bar{A}$.

b. Each $\theta_i = [\theta_{i,1}, \ldots, \theta_{i,\kappa}]$ is a κ-dimensional vector with values between 0 and y_{max}.

c. Let $n' = n_0$. This counter keeps track of the individual local search as opposed to n, which is the overall number of iterations. For RBFCNDG, $n' = n$.

LHS is a stratified random sampling technique that breaks down the sampling distribution into multiple regions to ensure full coverage of the range of the distribution in the most efficient manner. This sampling method is suggested by Regis and Shoemaker (2007) and has been used by Chen and Yang (2004) in simulating initial samples for their GA because it outperforms the Monte Carlo method. In particular, the LHS Matlab utility developed by Budiman (2004) is used for convenience.

The initial sample size is chosen to be 2κ so that the initial link allocations are stratified between the upper and lower halves of the 0 to 1 range. To deal with the constraints of the CNDP, the following step is employed.

Step 2. Scale the values up to the budget constraint. Although the MSRBF algorithm is designed to handle box constraints, it can also handle any functional constraints such as Eq. (3.4) as discussed in Regis and Shoemaker (2007).

a. Determine the linear combination $\sum_{a=1}^{\kappa} \theta_{i,a}^{\gamma} d_a$ where each d_a corresponds to the appropriate link cost construction coefficient. If this falls within the budget B, then let $y_i = \theta_i$.

b. Otherwise, let $y_i = \left(\sum_{a=1}^{\kappa} \theta_{i,a}^{\gamma} d_a\right)\theta_i / B$.

This method generates random solutions along or within the constraint for evaluation.

Step 3. For each iteration $1 \leq i \leq n'$, obtain the user equilibrium link flows x_i from Eq. (3.6) – (3.9) with the y_i using the Frank-Wolfe algorithm.

Step 4. Evaluate the total system travel times (TSTT) of each y_i in the set $S_{n'}$ by computing the upper level objective function value, $TSTT_i = \sum_{a \in A} c_a\left(x_{i,a}, y_{i,a}\right) x_{i,a}$.

a. The best solution found is stored as y_n^* with a best objective value of $TSTT_n^*$.

RBF's can now be generated based on the initial set of inputs $y_{i,a}$'s and outputs $TSTT_i$'s.

Step 5. For RBFCNDL, let Ω ≡ 0.95, so that fitness of candidate points are always weighted 95% by the objective value and 5% by the distance criteria.

a. RBFCNDG: Initiate a cycle path Ω for the purpose of alternating between a purely global search (Ω = 0) to a purely local search (Ω = 1). For the CNDP, the following path was used: Ω = {0.05, 0.75, 0.95, 0.97, 1}.

   i. Set a counter $\delta = 1$ such that $\Omega(\delta) = 0.05$.

Step 6. Estimate the coefficients of Eq. (5.2) using Eq. (5.3) for the *n'*-sample set. The *p* in Eq. (5.2) is a combination of polynomials of order *m*.

$$s(y) = \sum_{i=1}^{n'} \lambda_i (r_i^2 log r_i) + p(y), y \in \mathbb{R}^\kappa, r_i = \|y - y_i\| \tag{5.2}$$

The coefficients can be obtained by solving the system of equations in Eq. (5.3):

$$\begin{pmatrix} \lambda \\ \pi \end{pmatrix} = \begin{pmatrix} \Phi & P \\ P^T & 0 \end{pmatrix}^{-1} \begin{pmatrix} F \\ 0 \end{pmatrix} \tag{5.3}$$

Where

Φ_{ij} is $r_{ij}^2 log r_{ij}$, where $r_{ij} = \|y_j - y_i\|$ for $1 \le i, j \le n$;

P is an *n* x (κ + 1) matrix for 1st order polynomials where the first κ column elements are $y_{i,j}$, *j* = 1 to κ, and the last column is 1

F is the vector of $TSTT_i$'s.

Polynomials with order m = 1 are considered for evaluation because tests with higher order m = 2 shows worse results. Based on the framework by Regis and Shoemaker (2007), a well performing function is the thin plate spline function $\phi(r) = r^2 logr$ used in Eq. (5.2).

Step 7. Randomly generate T candidate points, z_j, $1 \leq j \leq T$. For RBFCNDL, random points are generated around the neighborhood of y_n^* using a vector of independent inverse normal distributions with mean value at $y_n^* = \{y_{n,1}^*, \dots, y_{n,\kappa}^*\}$ and standard deviation $\sigma = \{\sigma_1, \dots, \sigma_\kappa\}$. An initial value of $\sigma = 0.1y_{max}$ is used.

a. The number of candidate points $T = 100\kappa$ is considered (for a network with 10 possible link expansion allocations, T = 1000). For each z_j, $z_j = \max\left(\min\left(z_j, y_{max}\right), 0\right)$ to ensure feasibility; if the constraint is exceeded then unilaterally scale all the elements down to the constraint.

b. For RBFCNDG, simple uniform random sampling is used for the purpose of exploring the possible points in the space.

Step 8. For each candidate point z_j for $1 \leq j \leq T$, evaluate $s_{n'}(z_j)$. Additionally, evaluate a distance measure as defined in Eq (5.4). Store the maximum and minimum $s_{n'}(z_j)$ and $\Delta_{n'}(z_j)$ values.

$$\Delta_{n'}(z_j) = \arg\min_{1 \le i \le n'} \|z_j - y_i\| \tag{5.4}$$

Step 9. Evaluate the fitness criteria of each candidate point to be the next point using Eq. (5.5) such that the minimum is preferred.

$$W_{n'}(z_j) = \Omega(\delta) \frac{s_{n'}(z_j) - s_{n',min}}{s_{n',max} - s_{n',min}} + (1 - \Omega(\delta)) \frac{\Delta_{n',max} - \Delta_{n'}(z_j)}{\Delta_{n',max} - \Delta_{n',min}} \tag{5.5}$$

In this evaluation, the first term corresponds to the minimum interpolated *TSTT* using the RBF's and reinforces the local optimization. The second term represents the distance of each candidate point to the nearest point in the set $S_{n'}$, which reinforces the exploratory aspect of the global optimization.

Step 10. Let $y_{n'+1} = z_j$, and evaluate $TSTT_{n'+1}$ using Eq. (3.6) – (3.9) and (3.3).

a. Let $S_{n'+1} = \{S_{n'}, y_{n'+1}\}$

b. If $TSTT_{n'+1} \ge TSTT_n^*$ then no improvement has been made in that iteration. Let $\zeta = \zeta + 1$; else if improvement has been made set $\zeta = 0$.

c. If $\zeta = \zeta_{max}$, then let $\sigma = \sigma/2$ and $\rho = \rho + 1$. This assures convergence of the neighborhood search.

Step 11. Set $n = n + 1$. If $n = N_{max}$, stop. Else, if $\rho = \rho_{max}$, go to Step 1. Else if $\rho < \rho_{max}$, set $n' = n' + 1$ and go to Step 6.

a. For RBFCNDG, let δ = 5mod(δ + 1).

At the end of the algorithm, the minimum $TSTT^*_{N_{max}}$ obtained is the solution with the corresponding $y^*_{N_{max}}$. If the $TSTT^*_i$'s from the RBFCNDL are plotted by iteration, the result would be cycles of converging values that lead to restarts. For the RBFCNDG, the result would be cycles of highs and lows because of the alternating between global and local search. The highlights of the RBFCNDL algorithm are shown in FIGURE 5-1.

In terms of computational efficiency, both the RBFCNDL and RBFCNDG require up to N_{max} evaluations of Eq. (3.6) – (3.9) and $T(N_{max} - n_0)$ evaluations of the computationally cheaper candidate point interpolations and evaluations. Estimating the parameters of the interpolation function in each iteration is considered computationally cheap compared to the network optimization. A sample 1000-iteration run of the Sioux Falls network with 24 nodes, 10 investment links, 76 overall links showed that 95% of the computational time is allocated to the network optimization. Details of run time results are shown in the numerical tests in the next section.

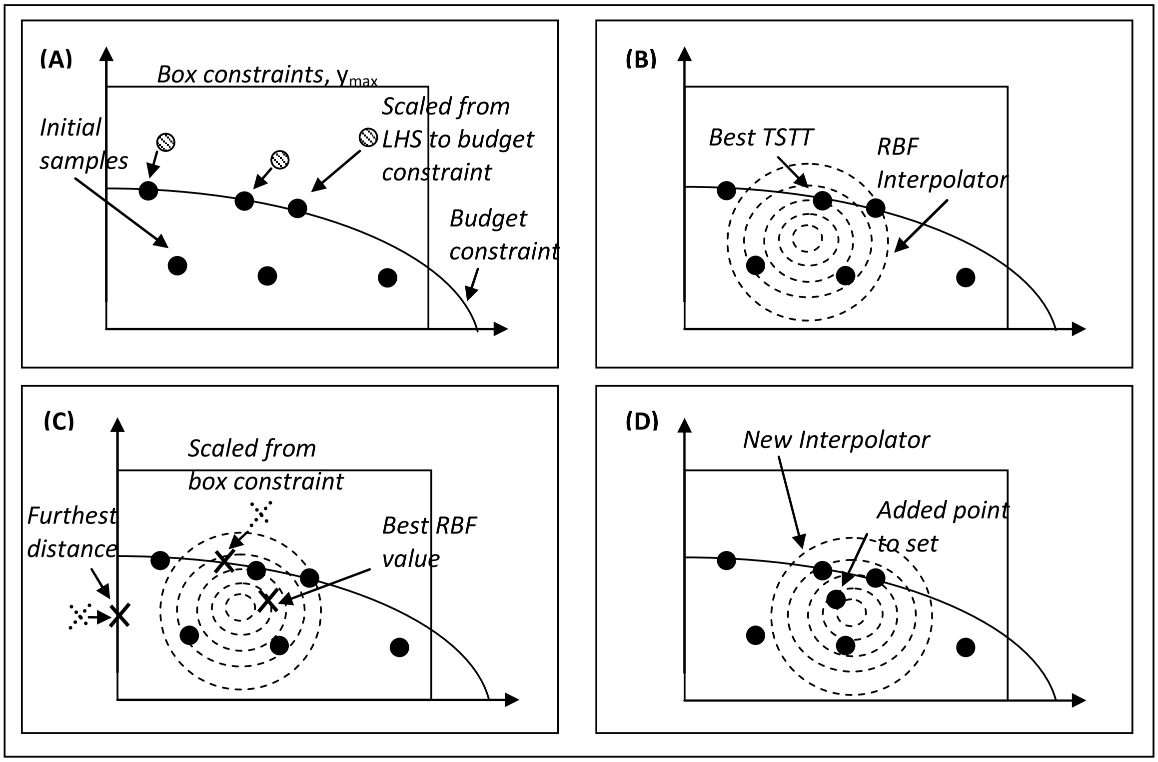

FIGURE 5-1. RBFCNDL for CNDP: A) Steps 1-2, B) Steps 3-6, C) Steps 7-9, D) Steps 5, 10, 11.

5.3 NUMERICAL TEST OF RBFCNDL

The RBFCNDL and RBFCNDG algorithms are compared to the GA approach used in Chen and Yang (2004). Further, for benchmarking we also compare these to the local heuristic Iterative Optimization Algorithm (IOA) as described by Yang and Bell (1998). While that algorithm works exceptionally well for the problem we examine for illustrative purposes, we do not believe it will be possible to apply to more complex and larger problems and it has the potential to get stuck in local optima.

The Sioux Falls, SD network with 24 nodes and 76 links (Chen and Yang, 2004) is used. The network data and parameters are presented in Appendix D. Five runs of each algorithm are made for comparison. In addition to the Sioux Falls network with standard OD flows, a more congested scenario with double the OD flows is tested to compare performance of the algorithms.

The Anaheim, CA network with 38 centroids, 416 nodes and 914 links (Jayakrishnan et al, 1994) is used to illustrate the scalability of the RBF methods compared to GA. For that network, 31 links are chosen as potential candidates for capacity expansion, making it a 31-dimensional network design problem.

5.3.1. Sioux Falls Network

The network parameters are based on the parameters referred to in Chen and Yang (2004). Ten links are considered for capacity expansion given a budget of 5500 units (equivalent of $5.5M).

The Iterative Optimization Algorithm (IOA) discussed in Section 3.2.1.5 involves iteratively fixing the upper and lower levels of the bi-level problem and solving one constrained optimization problem in each iteration. For the Sioux Falls network, the budget constraint is a quadratic constraint so the “fmincon” function in Matlab is used to solve the upper level for convenience. While IOA is an extremely fast heuristic, the solution is a Cournot-Nash equilibrium as opposed to being a more realistic Stackelberg equilibrium with leader-follower underlying behavior. The implication of this difference is that the solution assumes that the drivers on the road do not have perfect knowledge of where the agency is making the link improvements. The IOA approach is used with a tolerance of relative difference in the TSTT of 0.01 and Frank-Wolfe iterations up to 100 iterations. The converged TSTT after 3 iterations with a relative tolerance of 0.001 is 75.93, compared to a no investment TSTT of 101.20.

After running 5 sample runs of 1000 iterations of UE assignment for RBFCNDL, RBFCNDG, and GA (992 UE evaluations based on 30 generations with an initial 64 population sample and subsequent 32 evaluations per generation), the convergence rates of the 5 sample runs for each algorithm are plotted in FIGURE 5-2. The minimum, average, and maximum of the five runs for each are summarized in TABLE 5-1, as well as their average run times. The algorithms were run in Matlab 7.7.0 (R2008b) on a Windows XP Professional 2002 SP3, Intel Core2 Quad CPU with Q6600 @2.40GHz and 2GB RAM.

TABLE 5-1. **Optimal Value Comparison from 5 sample runs for RBFCNDL, RBFCNDG, and GA**

Obj (1000 iter)	IOA (benchmark)	RBFCNDL	RBFCNDG	GA (992 iter)
Min TSTT*	75.93	75.53	75.61	75.78
Avg TSTT*	75.93	75.53	75.64	76.01
Max TSTT*	75.93	75.54	75.67	76.85
Total Run Time (min)	--	41	105	33

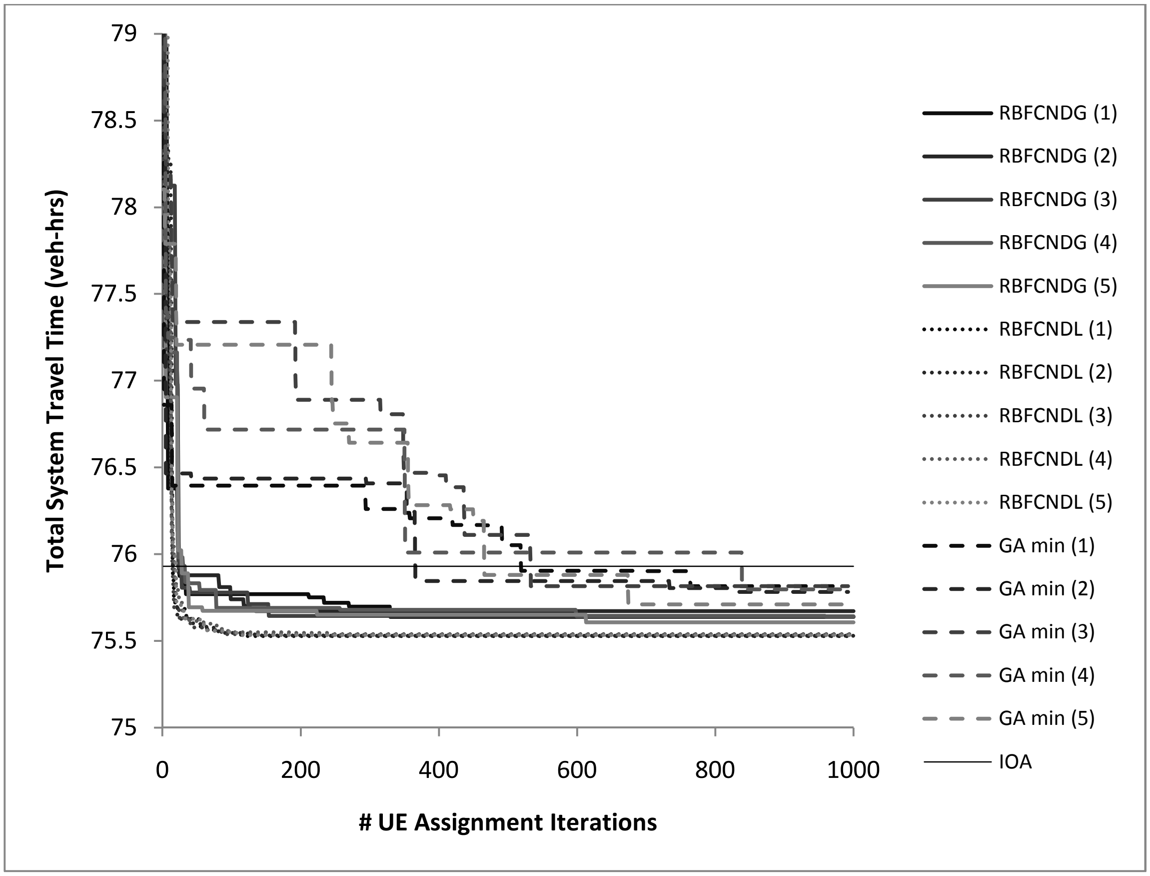

FIGURE 5-2. Convergence rates of optimal solution for standard Sioux Falls OD flows.

The figure shows the optimal values as functions of the number of iterations for the three methods being compared, with both RBF algorithms significantly outperforming the GA in convergence rate for all five runs. As mentioned above, the IOA result is included for benchmark comparison. All three algorithms obtain better results than the IOA heuristic within 1000 iterations (within 100 iterations for the RBF algorithms and within 800 iterations for GA).

The results show that the RBFCNDL algorithm converges extremely fast in terms of both the number of UE evaluations (within 100 iterations) and in the run time. The run time of the RBFCNDG algorithm is greater than twice that of the RBFCNDL because the sample set is never reset so the matrix used for estimating the interpolation coefficients increases in size with the number of iterations. This design prevents the RBFCNDG from being a suitable algorithm for large-scale problems. This conclusion verifies the conclusion made in Regis and Shoemaker (2007) for the problems they tested on.

The total run time of the RBFCNDL is greater than the GA because of additional computational time for interpolation and generating candidate points. However, the algorithm appears to reach convergence within 200 iterations, or less than 1/5 of the total run time. That means the RBFCNDL converges in less than 8 minutes compared to the GA, which does not reach the same converged values in 33 minutes.

The RBFCNDL and GA algorithms are also allowed to run to 8000 iterations and 249 generations (64 initial population with 248 generations of 32 evaluations each resulting in the same 8000 iterations of UE evaluation). The results are compared in TABLE 5-2. The best solution obtained by the RBFCNDL from one run up to 8000 iterations is 75.53, reached on the 194th iteration. The run time for the 8000 iterations of RBFCNDL is 325 minutes, although the optimal solution is found after approximately 8 minutes.

TABLE 5-2. Optimal Value Comparison from 8000 Iteration/249 Generation Run

	RBFCNDL (8000 Iterations)	**GA (249 Generations)**
TSTT*	75.53 veh-hr	75.67 veh-hr
Total Run Time	325 min	265 min
N*	194 iter.	7696 iter.
Run Time to N*	8 min	255 min

5.3.2. Sioux Falls with 2x OD Flows

In this more congested scenario, OD demand from the standard Sioux Falls network is multiplied by two. The IOA approach converges after 3 iterations (with a 0.001 relative tolerance) resulting in a TSTT = 1154.4. This value is in comparison to a no investment user equilibrium of TSTT = 1963.5.

The optimal solutions from running five samples of 1000 iterations of UE assignment for RBFCNDL, RBFCNDG, and GA are shown in TABLE 5-3. With more congestion, only the RBFCNDL reaches the IOA solution within 1000 UE iterations. The run times do not change by much under the more congested conditions. The convergence rates of these three methods are shown side by side in FIGURE 5-3. The solutions in TABLE 5-3 for GA are non-converged, whereas the RBFCNDL results are generally converged by the 200th iteration, as shown in FIGURE 5-3.

TABLE 5-3. Optimal Value Comparison from 5 sample runs, 2x OD Flow

Obj (1000 iter)	IOA (benchmark)	RBFCNDL	RBFCNDG	GA (992 iter)
Min TSTT*	1154.4	1154.2	1156.9	1157.9
Avg TSTT*	1154.4	1154.4	1157.6	1160.1
Max TSTT*	1154.4	1154.4	1158.6	1162.5
Total Run Time (min)	--	42	107	34

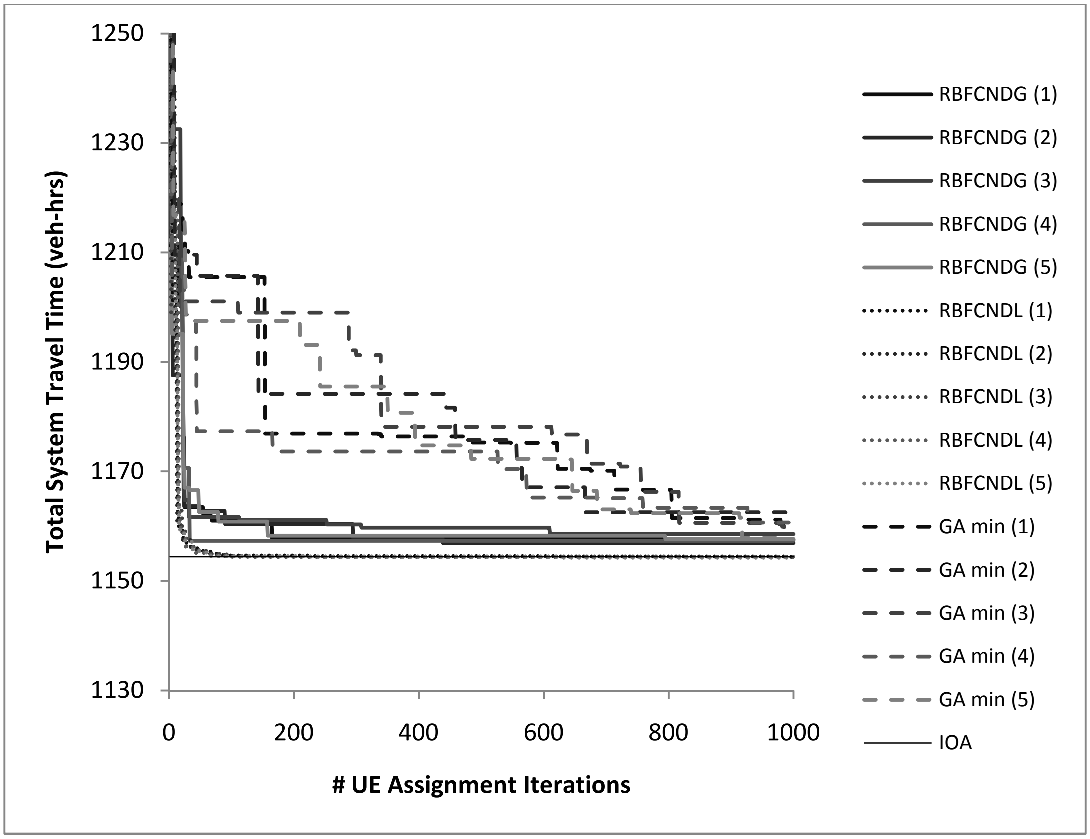

FIGURE 5-3**. Convergence rates of optimal solution for Sioux Falls 2x OD flows.**

5.3.3. Anaheim, CA Network

A larger network is used to illustrate the scalability of the algorithms. The Anaheim network data from 1992 is obtained from the Bar Gera website (2007), which was used by Jayakrishnan et al (1994). A map of the network is shown in FIGURE 5-4. In addition to the 416 nodes and 914 links, only 38 of the nodes are OD centroids that serve as sources and sinks for demand. OD flow is not allowed to use centroids as through nodes in order to prevent going onto the artificially constructed centroid connectors. With the same tolerance of 0.01 and 100 max iterations, the UE assignment with Frank-Wolfe results in an objective value of 21,433.9 veh-hr and a TSTT of 23,665.9 veh-hr.

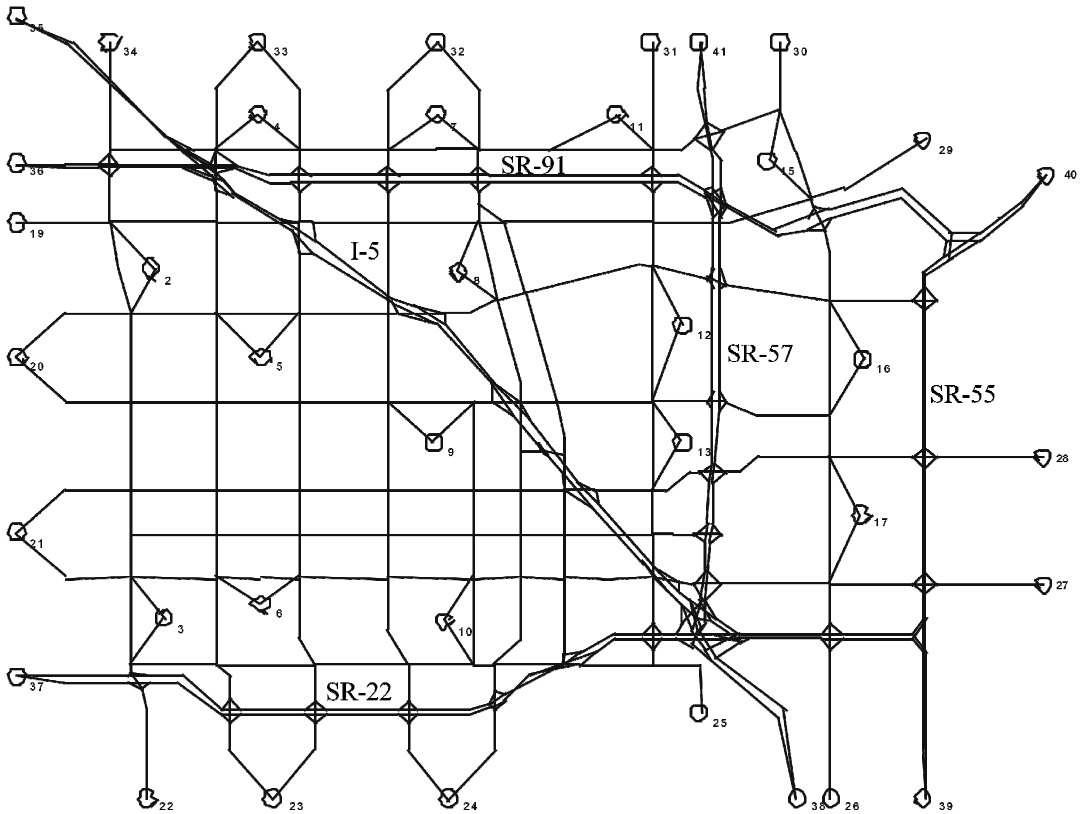

FIGURE 5-4. Anaheim network *(*Bar-Gera, 2007*)*.

Additional parameters were determined for this experiment with the CNDP since no such parameters are included in the data set. Similar to the Sioux Falls network, a subset of links are chosen as candidates for capacity expansion. TABLE 5-4 shows the set of 31 links chosen based on a criterion of selecting link candidates that are more than 1 mile long. With a capacity of 1800 veh/hr/lane (vphpl), a maximum of 7 lanes or 12,600 vph was assumed possible for each link. The y_{max} were determined by subtracting the existing capacity for each of the 31 links from the maximum 12,600 vph. The construction cost coefficients, d_a, were determined based on the y_{max} values: d_a = 2 if y_{max} = 3600, d_a = 3 if y_{max} = 5400, and d_a = 4 if y_{max} = 7200. After computing the minimum budget needed to build any one of the links to 7 lane capacity, that value 28,800 was doubled and rounded up to an eventual budget of *B* = 60,000.

Using these parameters, the IOA solution after 5 iterations is *TSTT* = 23,559.9 veh-hr, which has a savings of 106.1 veh-hrs.

The RBFCNDL and RBFCNDG algorithms are tested up to 1000 iterations while the GA algorithm is run up to 30 generations (64 initial population + 32 evaluations * 29 subsequent generations = 992 UE evaluations).

The convergence rates of each of the sample runs are compared in FIGURE 5-5. Since the results of the two sample runs clearly show the gap in performance between the RBFCNDL and the GA within the 1000 iterations, no further sample runs or further iterations are necessary. The RBFCNDL run time increases from approximately 40 minutes for the Sioux Falls network with 24 nodes and 10 investment links to approximately 2400 minutes for the Anaheim network with 416 nodes and 31 investment links.

The best results from two sample runs of each method are presented in TABLE 5-4 along with the IOA values. The best sample run from RBFCNDL has a TSTT of 23,555.6 veh-hr after the 1000 iterations while the RBFCNDG has a TSTT of 23,603.7 veh-hr. In comparison, the GA algorithm converges to an optimal TSTT = 23,578.2 veh-hr after the 30 generations. Only the RBFCNDL algorithm achieves a better TSTT than the IOA within 1000 UE iterations, obtaining its best solution on the 298^{th} iteration, which takes approximately 718 minutes compared to the 2062 minutes needed by GA for its 30-generation optimal solution. Let's refer to the iteration upon which the best solution is obtained as N^*. This test demonstrates both the weakness of the RBFCNDG algorithm for 31-dimensional problems and the strength of the RBFCNDL algorithm under the larger-scale setting.

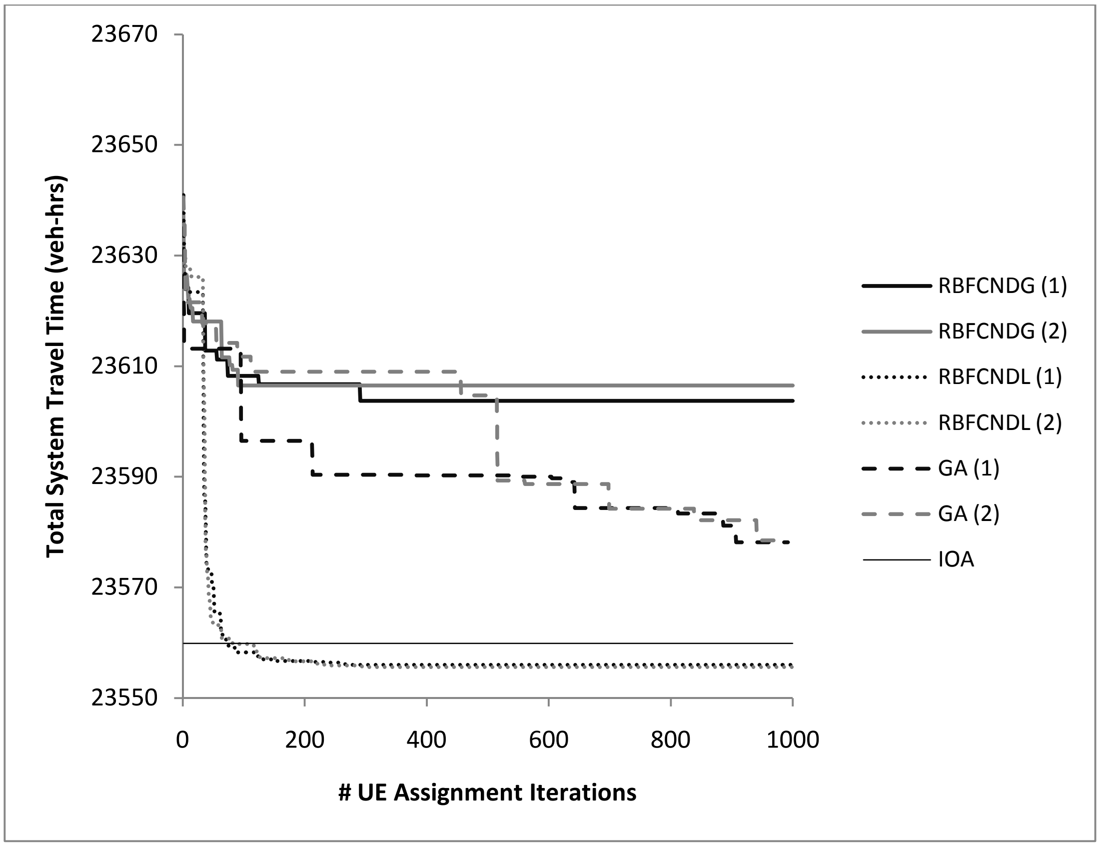

FIGURE 5-5**. Convergence rates of optimal solution for Anaheim, CA network.**

TABLE 5-4. Anaheim CNDP parameters and solutions

ID	Link #	y_{max}	d_a	IOA	RBFCNDL	RBFCNDG	GA
1	71	7200	4	0	40.6	162.7	616.5
2	110	5400	3	2608.63	2813.0	603.3	253.3
3	113	5400	3	3220.91	2956.3	1085.8	3373.3
4	127	5400	3	2571.77	681.7	1153.8	730.5
5	130	5400	3	1023.24	77.4	399.4	112.0
6	150	5400	3	150.29	433.9	81.3	583.0
7	172	5400	3	372.81	1533.0	654.5	750.7
8	272	5400	3	3878.55	3407.9	932.7	2102.3
9	301	5400	3	5400	7170.8	1099.8	3050.7
10	331	3600	2	43.07	265.1	126.3	813.9
11	369	3600	2	1117.62	817.1	218.3	828.6
12	414	7200	4	0	0	36.7	472.0
13	428	7200	4	0	0	517.3	190.9
14	436	7200	4	0	0	8.6	327.8
15	451	7200	4	0	0	223.3	4.0
16	483	7200	4	0	0	1627.3	125.9
17	528	7200	4	0	0	426.5	90.4
18	537	7200	4	0	34.7	831.1	352.8
19	558	7200	4	0	0	57.3	87.7
20	567	7200	4	0	23.0	1480.7	529.5
21	580	7200	4	0	10.7	154.2	449.0
22	584	7200	4	0	0	320.9	391.1
23	585	7200	4	0	0.1	88.0	391.2
24	588	7200	4	0	0	628.8	75.8
25	636	7200	4	0	0	212.7	11.8
26	648	7200	4	0	0	373.8	304.5
27	719	7200	4	0	0	802.3	455.5
28	749	7200	4	0	0	66.3	266.9
29	805	7200	4	0	0	391.1	618.2
30	834	7200	4	0	44.3	1221.9	44.8
31	891	7200	4	0	0	688.0	22.3
TSTT*	-			23,559.9	23,555.6	23,603.7	23,578.2
N*				-	298^{th} iter.	291^{st} iter.	886^{th} iter.
Budget				60,000	60,000	60,000	59,467.41
Total Run Time				-	2410 min	2538 min	2309 min
Time to N*					718 min	739 min	2062 min

In terms of scalability, the ratio of total run time for 30 generations of GA on the smaller Sioux Falls network compared to 1000 iterations of the RBFCNDL is 0.80. By comparison, the same ratio for the Anaheim network is 0.96. Clearly, the RBF interpolation time becomes negligible relative to the network optimization for large-scale networks. For the RBFCNDL, the ratio of "run time to N*" for the Anaheim to Sioux Falls network is 718:8.

By increasing the network size from 24 nodes and 10 expansion dimensions to 416 nodes and 31 dimensions, the convergence rate of the RBFCNDL increased 90 times. For the GA, convergence is not achieved for the Anaheim network, and it could take an estimated 18,500 minutes or 13 days to obtain the results for 250 generations on the same computing environment.

This example illustrates the potential of the multi-start local MSRBF algorithm as a much faster heuristic for solving network design problems with up to 31-dimensions and 416 nodes (RBFCNDL). More complex problems that are based on continuous network design elements, such as toll pricing, multi-objective applications, and real option dynamic programming computations for continuous network design investments can find potential value from this algorithm, as shown in Chapter 3 and Chapter 6.

5.4 MULTI-OBJECTIVE EXTENSION

As described in Chen et al (2006), the GA method used to solve the multi-objective problem incorporates a distance criterion for fitness in a population because a Pareto set could have multiple non-dominated solutions that cannot be evaluated for fitness using single-objective criteria. According to Fonseca and Fleming (1993), GA's are ideal algorithms for multi-objective optimization because they maintain a population of solutions each generation, which works very well for searching for approximate Pareto optimal sets.

The stochastic response surface (SRS) algorithm from Regis and Shoemaker (2007) already incorporates a distance criterion to encourage the search for a global solution. When evaluating all the candidate points using the interpolation function in each iteration, the SRS chooses a best fit solution based on a weighted combination of the best estimated objective value to a neighborhood search along with the furthest candidate point from the evaluated points. The MSRBF algorithm also maintains a set of solutions but instead of an evolutionary approach of updating that set, it continually adds new candidate points obtained from interpolation of the current set.

In the proposed algorithm, the method is altered in several ways. First, the multi-start local MSRBF method is used to construct a set of candidate points by simultaneously interpolating all the objectives in the objective vector.

When local convergence is reached in the multi-start local MSRBF, the algorithm resets with a new initial sample set. This approach is modified so that now the reset would still occur, but the Pareto set would be kept.

Third, the distance criterion from the RBF method for choosing the optimal candidate point is modified to obtain the maximin distance from the existing approximate Pareto set's objective vectors instead of from the existing local MSRBF set of solutions.

Fourth, the choice of the best solution to randomly generate the neighborhood of new candidate points is determined by sorting the current Pareto set and finding the objective vector that is furthest from the other vectors. The generalized algorithm is outlined here.

5.4.4. Generalized Multi-Objective RBF (MO-RBF) Algorithm

Step 0. Initiate an empty Pareto set Ω_0. Initiate neighborhood search tolerances, r_{max} and ζ_{max}. Initiate $n = 0$ and $n_{cycle} = 0$.

Step 1. Initiate ρ_M as a vector of standard deviations for a normally distributed random search around the neighborhood of a chosen vector. Generate n_0 initial variables using Latin hypercube sampling within existing constraints. Let $n = n + n_0$. Let $n_{cycle} = n_0$. Set an initial counter $\zeta = 0$ and $r = 0$.

Step 2. For each n^{th} sample, obtain the objective value vector and evaluate its non-dominance criteria relative to the existing Pareto set Ω_{n-1} and update the Pareto set Ω_n.

Step 3. Sort Ω_n by the first objective to obtain $\Omega_n^{'}$ and determine the vector that has the greatest distance from its adjacent neighbors. For the vectors at the ends of the set, use the distance from their first and second neighbors instead. Select the point using the criteria in Eq. (5.6). For $|\Omega_n| < 3$, select the first point instead.

$$\max_{F_i} \Delta = \arg\min_{i\in|\Omega_n'|}(\|F_i - F_{i-1}\| + \|F_i - F_{i+1}\|) \quad (5.6)$$

Alternatively, the Manhattan distance can be used instead of the Euclidean distance to emphasize the search around points that are more sensitive to trade-offs between the objectives.

Step 4. Fit or update a radial basis function (RBF) with Eq. (5.2) by solving Eq. (5.3) for each objective interpolator s^k for k objectives in a vector.

Step 5. Generate P candidate points using randomly sampled points distributed normally around the chosen point from Step 3 with mean of zero and standard deviation of Iρ_M. Evaluate each candidate point with Eq. (5.2) for each of the k objectives.

Step 6. Construct a temporary non-dominated set ϕ_p from the candidate points, sort it to obtain ϕ_p', and select the point that satisfies Eq. (5.7). Similar to Step 3, the end points would use their two closest neighbors instead, and Manhattan distance can be used instead of Euclidean distance.

$$\max_{s_i} \Delta = \arg\min_{i \in |\psi_P'|}(\|s_i - s_{i-1}\| + \|s_i - s_{i+1}\|) \quad (5.7)$$

Step 7. Update y_{n+1} to be the point that satisfies Eq. (5.7). Evaluate the objective vector given the new point.

Step 8. Update the Pareto set Ω_{n+1} and sort it. Set $n = n + 1$. If no improvement to the Pareto set is made, set $\zeta = \zeta + 1$. If $\zeta > \zeta_{max}$, $\rho_M = \rho_M/2$, $\zeta = 0$ and $r = r + 1$. If $n = N_{max}$, end the algorithm. If If $r \leq r_{max}$, go to Step 5, else go to Step 1.

5.4.4.1. Convergence

The output of this algorithm is the approximate Pareto optimal set of an objective vector after N_{max} iterations and the set of y's corresponding to each solution. The proof of probabilistic convergence for a simple objective problem provided by Regis and Shoemaker (2007) does not apply to this algorithm because of its multiple objectives. However, Schütze et al (2008) proves the following theorem (presented as Theorem 3.2 in that paper):

Theorem 5.1. *Given a multi-objective problem F:* $\mathbb{R}^M \rightarrow \mathbb{R}^k$*, where F is continuous, and let* $Q \subset \mathbb{R}^M$ *be a compact set and* $\epsilon \in \mathbb{R}_+^k$*. Further let:*

$$\forall y \in Q, \forall \eta > 0: \; P\big(\exists l \in \mathbb{N}: P_l \cap B_\eta(y) \cap Q \neq 0\big) = 1 \tag{5.8}$$

Then an application of any basic stochastic search algorithm, where the existing Pareto set is compared to each new solution and updated, leads to a sequence of archives $\Omega_n, n \in \mathbb{N}$*, such that there exists with probability one an* $n' \in \mathbb{N}$ *such that* Ω_n *is an* ϵ*-approximate Pareto set for all* $n \geq n'$*.*

Where P_l is the set of all populations up to the lth iteration and $B_\eta(x)$ is a region of radius η centered around a point x.

Proof. The detailed proof is presented in Schütze et al (2008), and essentially follows these logical steps: 1) acknowledging that the neighborhood of an immediately prior Pareto set should always be a subset of the neighborhood of the current Pareto set; 2) bounding the space of the permissible insertions to the current Pareto set; then 3) showing by contradiction that there almost surely exists a finite iteration $n' \in \mathbb{N}$ such that for all future iterations $n \geq n'$ the Ω_n is within ϵ distance to the true Pareto optimal set.

The MO-RBF algorithm requires the problem to be continuous, the domain to be a compact set, and includes the Pareto set update in Step 2. Therefore the convergence theorem applies to the proposed algorithm, which means that it would almost surely converge to the true Pareto optimal set as the number of iterations approaches infinity for a finite tolerance.

5.5 COMPARISON OF MO-RBF FUNCTION EVALUATION CONVERGENCE

Quantitative performance measures for multi-objective algorithms generally need to be at least binary in nature, as shown in Zitzler et al (2003). They define a binary ϵ-indicator for measuring one approximate Pareto set against another for general problems that do not have a known solution. Deb et al (2002) point out that those problems with known solutions can have more precise performance measures to evaluate algorithms with. They compile a number of problems with known solutions to test their NSGA-II algorithm against other existing methods. In particular, there is a problem with a Pareto set that is known to be both non-convex and non-uniformly distributed, which they call ZDT6. Its similar properties to Pareto sets for multi-objective network design problems make it an appealing test problem to compare the algorithms' effectiveness. The problem is shown in Eq. (5.9) – (5.11) and depicted in FIGURE 5-6.

$$f_1(x) = 1 - \exp(-4x_1)\sin^6(6\pi x_1) \quad (5.9)$$

$$f_2(x) = g(x)\left[1 - \left(\frac{f_1(x)}{g(x)}\right)^2\right] \quad (5.10)$$

$$g(x) = 1 + 9\left[\frac{(\sum_{i=2}^{10} x_i)}{9}\right]^{0.25} \quad (5.11)$$

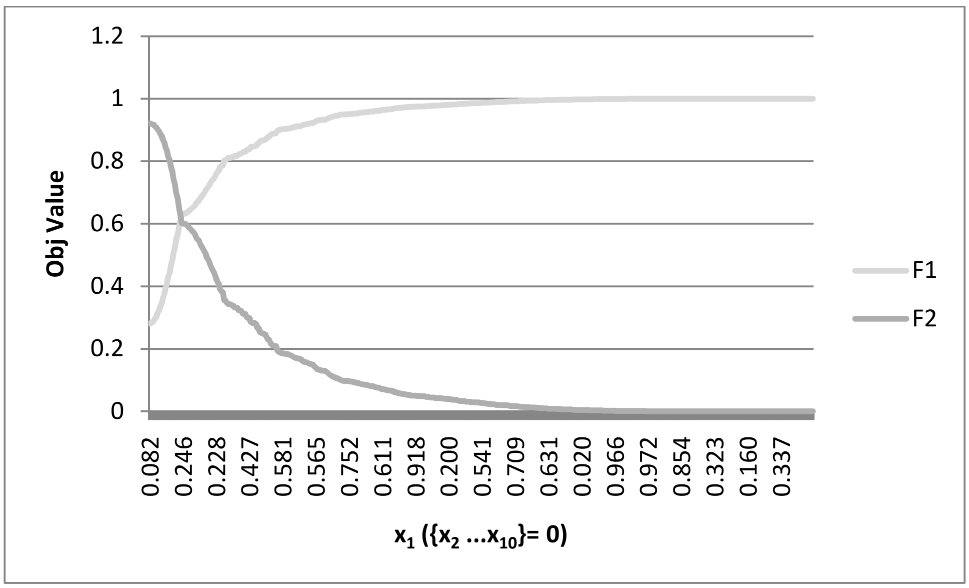

FIGURE 5-6. Objective Values as Function of x_1.

The problem is to minimize f_1 and f_2, where $x \in \mathbb{R}^{10}$ with box constraints [0,1]. The true Pareto optimal set lies along the $x_1 \in [0,1], x_i = 0$ for *i* = 2, 3, … , 10. The solutions presented in Deb et al (2002) compare two performance measures: a convergence criteria and a diversity criteria. The convergence criteria is the average distance between each solution and its closest point in the true Pareto optimal set. The diversity criterion is the average of the absolute value of the deviations from the mean. The values are obtained from 25,000 function evaluations.

It turns out that running MO-RBF on Eq. (5.9) – (5.11) just for one-fourth the number of iterations, or 6,250 iterations, already results in a better convergence criteria than all the other methods obtained from the full 25,000 iterations. The parameters are set to n_0 = 21, ρ_{M0} = 0.1*Y_{max}, r_{max} = 5, and ζ_{max} = 10, and P = 1000. TABLE 5-5 summarizes the results from Deb et al with the MO-RBF results from 6,250 and 25,000 iterations. The 6,250 iteration approximate set has unique 90 solutions in it, while the 25,000 iteration set has 181 unique solutions.

TABLE 5-5. Comparison of MO-RBF with other Algorithms for ZDT6

Algorithm	Mean Convergence	Variance Convergence	Mean Diversity	Variance Diversity
NSGA-II (real)	0.296564	0.013135	0.668025	0.009923
NSGA-II (binary)	7.806798	0.001667	0.644477	0.035042
SPEA	0.221138	0.000449	0.849389	0.002713
PAES	0.085469	0.006664	1.153052	0.003916
MO-RBF (6,250 iter)	0.0019	0.000009	1.2047	0.000932
MO-RBF (25,000 iter)	0.0023	0.000010	1.1022	0.000129

The results clearly show that MO-RBF has an order of magnitude better convergence than any of the other algorithms presented in Deb et al (2002). FIGURE 5-7 shows a comparison of the converged results with the NSGA-II and SPEA obtained from Deb et al.

Taken in consideration with the results from Section 5.3, these results show that the MO-RBF algorithm is a much faster heuristic than GA for complex, large-scale multi-objective network problems.

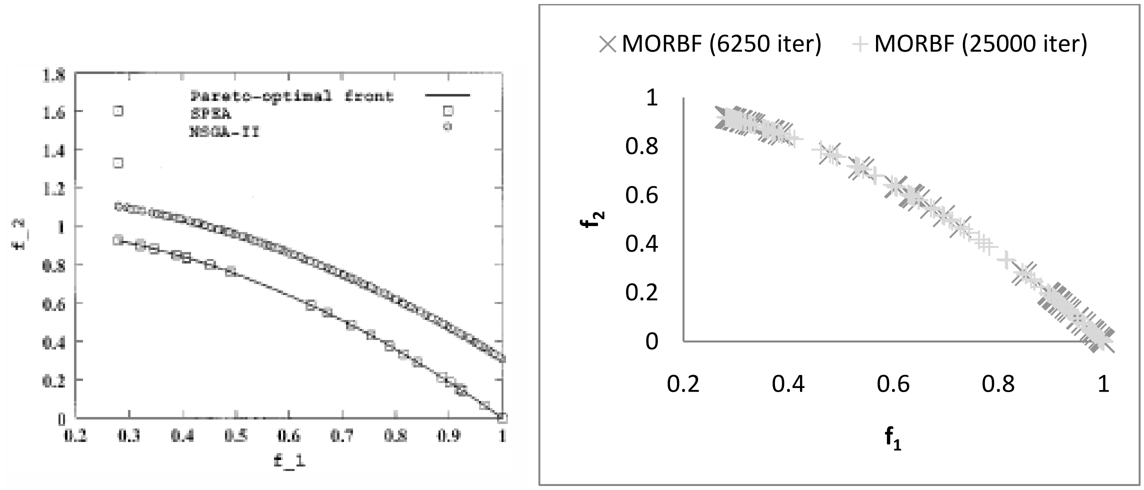

FIGURE 5-7. Comparison of MO-RBF decision space to NSGA-II and SPEA from Deb et al (2002).

5.6 DISCUSSION

Some relatively recent developments in constrained global optimization theory using radial basis function interpolation are applied to continuous network design-based models and extended to handle multi-objective problems. The continuous network design problem is considered because local heuristics are already available with which to compare the results. In addition, it is similar to many other network design problems and also serves as a basis for more complex network models, making it an excellent candidate for testing and extending the algorithms.

In Section 5.1 – 5.3, three experiments were conducted: the Sioux Falls network with standard OD flows; the same network with double the OD flows to test performance under a more congested scenario; and an experiment with the Anaheim, CA network to compare the scalability of their performances for problems with up to 31 dimensions. Parameters for the CNDP were developed for the Anaheim network to experiment with. The multi-start local MSRBF algorithm for CNDP (RBFCNDL) appears to converge faster than both the global MSRBF algorithm for CNDP (RBFCNDG) and a genetic algorithm in all the sample runs conducted.

One advantage that GA has over RBFCNDL is that it can handle both continuous and discrete decision variables. The RBF-based methods only deal with continuous decision variables at this point.

In Section 5.4 – 5.5, an RBF global heuristic is modified and a new multi-objective global heuristic, MO-RBF, is proposed as a fast converging heuristic for any multi-objective problem with computationally expensive objective functions. The algorithm is compared to existing multi-objective algorithms for a simple function with non-convex and non-uniformly distributed solutions and shown to outperform in terms of the number of objective function evaluations.

The RBFCNDL algorithm is applied to the NODP model in Chapter 3 to obtain an approximate optimal solution for designing a network to maximize real option value. The MO-RBF algorithm is used in Chapter 6 to solve the robust toll pricing problem and to evaluate the value of flexibility in switching between multiple seasons or regimes of link failure likelihood.

"Failure comes only when we forget our ideals and objectives and principles." – Jawaharlal Nehru

CHAPTER 6 FLEXIBLE ROBUST TOLL PRICING WITH MULTI-REGIME NETWORK DEGRADATION

This chapter is a culmination of many of the topics and themes discussed in earlier chapters about flexible transportation network management. Toll pricing is introduced as a network control that can manage the robustness of a network under stationary stochastic capacity. Robustness is managed using a Pareto optimal set of toll prices reflecting different degrees of expected social welfare and variance in that welfare for travelers in the network. This mean-variance toll pricing model is solved with the MO-RBF algorithm developed in Chapter 5.

The state of the network, defined as its regime, is allowed to shift from one distribution to another. An example of this is in blizzard seasons, where the weather forecasts may exogenously dictate whether a local region has a low chance of snow (resulting in low probability of link degradation due to snow) or has a blizzard warning (leading to a high probability of link degradation and a change in the distribution to reflect that).

The value of flexibility discussed in Chapter 2 is illustrated in a simple switching option setting where the Pareto optimal set of toll prices are allowed to adapt to a new regime. If costs were highly asymmetric, then more complex real option solution methodologies would be needed. Since the cost of setting a toll price is assumed negligible with respect to the change in social benefit that it can incur, it can be evaluated with a simple switching option valuation by taking the maximum of the two modes of operation.

Although applied to robust network toll pricing in an urban setting, the concept introduced in Chapter 6 can be applied to any network setting where flexible robustness can be achieved by allowing a multi-objective solution sets to adapt to new regimes.

6.1 INTRODUCTION

As the preceding chapters have shown, transportation planners and operational managers cannot focus solely on improving the efficiency of their networks for transporting people and goods; they must also heed other objectives such as minimizing environmental impacts or minimizing the sensitivity of system performance to uncertainties in supply and demand. Uncertainties in supply can arise from random incidents, such as an accident, a flood that closes off a roadway or a power failure that affects train operations. Many other causes of uncertainties exist – for example gradual road deterioration or unexpected events related to security threats.

One of the strategies available to network managers for tackling congestion is network toll pricing. By placing a toll on particular links or cordons in a network, it is possible to redirect traffic to reduce congestion throughout a network. There have been a number of successful implementations of pricing schemes in cities such as Singapore, London, and Stockholm (Tsekeris and Voβ, 2008).

Li et al (2007) propose using toll pricing as a strategy to manage uncertainty in a network with stationary stochastic OD demand and link capacity. Their approach is based on maximizing travel time reliability in a dynamic user equilibrium setting. Their strategy deals only with a single, static set of uncertainties. In other words, the underlying parameters for the probability distributions of the demand and capacity are static even if the network setting is dynamic. By defining multiple regimes instead, i.e. different threat levels in security or fire/hurricane seasons, it is possible to exploit information from exogenous sources for a more flexible network that responds to such information.

In the supply chain literature, Snyder et al (2007) describe a multiple regime setting for inventory management with risk pooling, where the probability distributions for demand can be subject to change in the long term. However, there has been no research examining how to evaluate toll pricing as a strategy to actively manage a network against multiple regime uncertainties. This chapter introduces two key contributions:

- Toll pricing is proposed as a strategy for managing uncertainty under multiple regimes; because of the negligible costs to set toll prices, it can be shown to be a simple switching real option. Instead of maximizing reliability, the multi-objective robust mean-variance formulation for toll pricing is used to offer the network managing agent greater control over uncertainty. An epsilon indicator is used to quantify the difference between two Pareto sets to determine the value of flexibility to switch Pareto sets in response to changes in the regime.
- The link capacity degradations are modeled with multivariate Bernoulli distributions for occurrences and independent uniform random distributions for the magnitude of loss; a simulation method that transforms the distributions into equivalent multivariate Normal distributions is shown to model direct correlations between multiple links.

A literature review is presented in Section 6.2, which ends with a discussion of flexible robust network toll pricing as a real option strategy. It leads up to the proposed simulation model formulation and an application of the MO-RBF solution algorithm from Chapter 5 in Section 6.3, followed by a numerical test and results in Section 6.4 where the value of multi-regime flexibility is explored empirically. Future research is discussed in Section 6.5.

6.2 LITERATURE REVIEW

6.2.1. Network Toll Pricing

Because of the abundance of literature on toll pricing, it would be out of scope to present an exhaustive review here. Recent extensive reviews of network toll and congestion pricing can be found in Small and Verhoef (2007) and in Tsekeris and Voβ (2009). Instead, relevant research leading up to the development of the proposed model is presented here.

The concept of using marginal cost pricing has been around for many years. Selfish driver behavior in congestion under user equilibrium tends to lead to a sub-optimal total travel cost in a network because drivers ignore the cost of externalities. In marginal cost pricing theory, the optimal flow is where the marginal cost and demand are equal. This equilibrium point can be reached by artificially placing a cost on a road to internalize the externality for the driver. For a detailed theoretical understanding of the principle, refer to Yang and Huang (1998).

While the theory sounds simple enough to implement, there are many complicating issues from both a theoretical and practical standpoint. Some theoretical issues have been pointed out in network level toll pricing: queuing, elastic demand, and spatial and welfare equity. Yang and Lam (1996) introduced a bi-level network toll problem that explicitly incorporates queuing in the formulation. Three different objectives were recommended: minimizing total cost, maximizing revenue, and maximizing the ratio of revenue to cost. Yang and Bell (1997) developed a similar bi-level toll problem but included elastic demand as well. They noted that the model that included both queuing and elastic demand applied to a subset of the links was not guaranteed to produce a feasible solution. In addition, because demand is elastic, cost minimization is not a reasonable objective. Instead, the following three objectives were proposed: maximizing total realized demand, maximizing consumer surplus, and maximizing total revenue.

Yang and Meng (1998) extended the explicit queue formulation into the space-time extended network to include departure time in addition to route choice and toll pricing. Subsequent static network models such as Yang and Zhang's (2002) only consider elastic demand without queuing while extending the problem to include multiple user classes and spatial equity. In their case, they use social welfare as the maximizing objective value. Social welfare is defined as the consumer surplus plus the profit from the toll pricing.

The problem discussed in Yang and Bell (1997) about a subset of links hints at a bigger practical issue. Although marginal cost pricing theory is meant to consider every link in a network for tolling to achieve the system optimal condition, this "first best" approach is not realistic. It is often more practical to toll only a specific subset of links, although this could lead to solutions that do not reach the system optimal value. Verhoef (2002) and Shepherd and Sumalee (2004) developed heuristics to simultaneously estimate both the optimal locations and the prices for tolls on a subset of links using heuristics such as simulated annealing and genetic algorithms. On an operational level however, generally the locations of the tolls are already determined and the only problem is to obtain the optimal prices. A bi-level scheme incorporating elastic demand such as the one proposed by Yang and Zhang (2002) can solve such a problem using a stochastic optimization heuristic such as simulated annealing.

More recently, Chen et al (2006) proposed a planning level Build-Operate-Transfer (BOT) network design problem that obtains optimal second-best toll prices and capacity improvements while accounting for uncertainty in demand. They showed that a robust approach can be achieved for toll pricing using a mean-variance multi-objective model that maximizes expected profit or social welfare while minimizing the variance.

6.2.2. Robust Network Design

Snyder et al (2006) pointed out the importance of accounting for disruptions during the design of a supply chain network so that it can perform well even after a disruption. Whereas reliability is a probability measure for the likelihood of operating at 100%, robustness is the ability to maintain a given level of output after a failure. Ukkusuri et al (2007) differentiated robust optimization from stochastic programming as follows: stochastic programming accounts for uncertainty by optimizing the first moment of the distribution of the objective function, i.e. the expected value, while robust optimization also considers higher moments of the probability distribution.

Robust optimization was introduced by Mulvey et al (1995). Three common methods have been developed to handle robust optimization: the mean-variance method, minimizing the coefficient of variation, and the min-max approach. The mean-variance method has an objective function composed of a weighted sum of the expected value and variance of the performance and was developed by Markowitz (1987) for different purposes. The min-max approach minimizes regret in scenario planning, or in other words, minimizes the worst case scenario that could occur. All three methods are discussed in detail by Yin (2008).

There are a number of applications of robust network design in transportation. Chen et al (2006) proposed a robust continuous network design problem with toll pricing using the mean-variance method, and obtained the Pareto optimal frontier using a genetic algorithm.

Ukkusuri et al (2007) formulated the discrete network design problem as a "robust network design problem" using a mean-variance approach with fixed weights. Sharma et al (2009) extended that work to obtaining a Pareto frontier for undetermined weights using genetic algorithm. Lou et al (2009) considered the robust discrete network design problem as a mathematical program with complementarity constraints using a cutting plane method to solve the min-max formulation.

Ordóñez and Zhao (2007) applied column generation to solve the min-max formulation of a robust transshipment continuous network design problem. Mudchanatongsuk et al (2008) followed up with a discrete version of the model.

Karoonsoontawong and Waller (2007) applied the robust approach to the continuous network design problem with dynamic traffic assignment using cell transmission. The formulation is based on the mean-variance approach. The approach is limited to a single destination problem.

Yin (2008) proposed a robust approach to optimal traffic signal timing using all three methods described above, but only looked at an isolated fixed-time signalized intersection. Further, Yin et al (2009) applies scenario based, sensitivity based and min-max optimization based approaches to general road network improvement problems under demand uncertainty.

Essentially, robust optimization provides a mechanism for tradeoffs between maximizing efficiency and mitigating risks of loss. Decision-makers are provided with a tool that allows them to adjust the desired performance of the system based upon their preferences, or upon those imposed by the environment. While this approach gives more options to the long term planner as the literature suggests, it can be an even more important tool for providing flexibility to the short term operations manager. For example, a transportation network located on the Gulf coast might be subject to multiple flood seasons in a year, and a transportation manager could more effectively make use of a flexible robust toll system that adapts its prices based on information regarding the flood season and associated uncertainty regime.

6.2.3. Modeling Capacity Uncertainty

For the purpose of modeling uncertainty with a set of scenarios, it does not make sense to model link failures as binary states because it is not realistic and can also lead to accessibility issues in the network equilibrium convergence (isolated nodes with no link connections). Instead, continuous degradation is more appropriate and can represent more realistic cases of link failure incidents. Du and Nicholson (1997) concluded that ignoring dependencies between link components could have significant effects on the results.

Chen et al (2002) addressed capacity reliability by modeling the link capacities with multivariate normal distributions. They use an orthogonal transform method developed by Chang et al (1994) to convert Monte Carlo simulations of the marginal distributions to that of a joint distribution. The method empirically converts the correlation matrix of the distributions to a standard multivariate normal distribution, and then takes the inverse standard normal of the orthogonal matrix with simulated marginal distribution values. If two uniform distributions representing the cumulative distribution of two random variables are converted to equivalent normal distributions for simulation, the correlation matrix would need to be transformed. A nonlinear transformation is provided in Der Kiureghian and Liu (1986) and shown in Eq. (6.1).

$$\rho_{0,ij} = \rho_{ij}\left(1.047 - 0.047\rho_{ij}^2\right) \tag{6.1}$$

However, the method shown in Chang et al (1994) does not include mention of transforming the correlation matrix of two Bernoulli random variables.

Lo and Tung (2003) argued that the correlated methods are too complex for larger, more realistic networks and instead chose to model the link capacities as independent uniform random distributions. Siu and Lo (2008) extended the model to having both stochastic capacity and demand.

Sumalee and Watling (2003, 2008) propose a failure model based on correlating the causes behind the failures at links instead of at the links themselves. This method was developed because simulating multi-dimensional distributions for all the links in a network was too complex. Their proposed approach keeps the conditional link degradations independent from each other but still maintain a degree of correlation through the causes, such as the presence of “snow” or “high winds”.

For small numbers of causes, this may relieve the problem of having to resolve correlation, but the problem of multi-dimensional distributions still exists if many causes are considered jointly. Moreover, it can be argued that these causes are generally exogenous factors that are out of the control of transportation planning or operational agents. For example, under Sumalee and Watling’s method, the causes “snow” and “high winds” may have correlation with each other and independent degradation conditioned upon the joint occurrences, but it doesn’t make practical sense for a toll managing agency to model the probability of there being snow on a particular day. On the other hand, the agency can look at the correlated link failure scenarios given that there is snow and high winds on a particular day as a specific regime.

In the insurance industry, it is common to model a portfolio of risks with a multivariate Bernoulli random variable to indicate that claims have occurred, and to model the claim amounts as positive random variables (Frostig, 2001). However, even multivariate Bernoulli random variables can quickly escalate in complexity with the size of the dimension (Teugels, 1990).

One growing method that is adopted by the financial and insurance risk industry is the use of copulas, which are multivariate distributions whose marginals are uniform in the interval [0,1]. They are designed to map marginal distributions to a joint distribution while maintaining the dependency of the variables within its structure, and have been extremely effective in multivariate data analysis. According to Sklar's Theorem, given two marginals of any continuous cumulative distribution F(x) for Pr(X≤x)and G(y) for Pr(Y≤y), there exists a copula such that the joint distribution H(x,y) = C(F(x),G(y)) (Nelsen, 2006). However, copulas require at least one parameter, often not the same as the Pearson correlation coefficient (unless it is a Gaussian copula), and so it may need estimation for the research at hand. For a discussion of copulas applied to civil engineering systems see Singh, Jain and Tyagi (2007).

Curtis et al (2006) overcame this problem of simulating multi-dimensional Bernoulli variables when they derived a method to sample multivariate Bernoulli random variables by transforming them into normal distributions. They showed that if the mean and variance of the Bernoulli variables are treated as equivalent normal distributed parameters, there exists a threshold value τ such that Pr(X>τ) = Pr(Z=1), where Z is the Bernoulli variable and X is the transformed normal distributed variable with the same mean and variance. The τ can be obtained with Eq. (6.2).

$$\tau = \mu + \sigma\sqrt{2}\,\mathrm{inverf}(1 - 2\mu) \qquad (6.2)$$

Here the μ and σ are the mean and standard deviation of a Bernoulli variable. By converting the covariance matrix to an equivalent matrix for the normal distribution, random samples can then be drawn. In the new covariance matrix, the diagonals are kept the same, and the non-diagonals are determined from a correlation coefficient ρ_0 such that the relationship in Eq. (6.3) exists for each *i,j* pair.

$$\Pr(Y_i > \tau_i, Y_j > \tau_j; \rho_0) = \Sigma_{ij} + \mu_i\mu_j \tag{6.3}$$

In generating the values for the proposed model, the bivariate normal distribution is solved with the mvncdf function in MATLAB R2007b. A secant method bounded by [-0.9999,0.9999] is used to find the root value of ρ_0.

Therefore, the method proposed by Curtis et al provides a simpler simulation approach for multivariate Bernoulli distributions for every link in a network. Correlations can be maintained at the link level, while causes can still be considered by the use of multiple regimes and representing different probability distributions. Continuing the example discussed earlier, the method proposed here would look at the cases of snow, high winds, both, and neither, where each case can have correlated link failure occurrences. The details of the proposed stochastic capacity model are presented in Section 6.3.

6.2.4. Flexible Network Design

When accounting for uncertainty in network design, one approach is to provide solutions that can adapt to new information. Such a flexible solution is different from robust or reliable strategies which focus on controlling a static probability distribution. Flexible strategies are applied to exploit uncertainty, as shown in the prior chapters. A detailed review is provided in Chapter 2.

For projects that have highly asymmetric costs (such as an irreversible capital investment versus deferral) where the value of the project depends upon an underlying non-stationary stochastic process, the solution methods can be complex. Kulatilaka and Trigeorgis (2001) illustrates this with a simple abstract example of a switching option between a starting Mode A and an alternative Mode B. Under stochastic value conditions modeled as a decision tree, if the decision-maker does not have the option to switch from Mode A to Mode B then they would be subject to the outcomes of uncertainty. If they have an option to switch to Mode B associated with some fixed switching cost, then there are compound interactions that take place. If, on the other hand there are no costs to switch, then it is simply a case of option additivity that involves choosing the best mode in each branch of the decision tree.

It turns out toll pricing as a day-to-day tactical network design strategy has negligible costs to changing operating mode, especially compared to other costlier strategies such as long term capacity expansion. Since that is the case, toll pricing fits into the simplest category of option switching, which means that the value of flexibility can be obtained simply by computing the best set of toll prices at each branch, or regime in the case of this research, in the decision tree.

Following this same argument, the strategy that Li et al (2007) proposes in terms of minimizing unreliability would only determine how to obtain the best value at each branch of the decision tree. We propose to take a step further and show that toll pricing is an ideal strategy not only for managing uncertainty but for maintaining a flexible system under uncertainty because of the relatively negligible costs of changing operating mode (toll costs). In addition we can quantify the value of this flexibility by comparing it to a fixed initial toll design that does not have the same flexibility to respond to changing conditions.

6.3 PROPOSED NETWORK DEGRADATION SIMULATION MODEL

While the robust toll pricing problem can be generalized to different modes and transport networks, the focus here is on a static traffic road network G(N,A) in a given peak period. A dynamic network is not applied because the proposed toll pricing strategy for flexibility is tactical in natures, meant to respond to exogenous information that reflects a change in regime for a particular day and not as an operational strategy for minute to minute changes in stochastic demand. Origin-destination (OD) demand is present from every node in N to every other node, and is assumed to be a function of the travel cost of the path. An exponential function is used similar to Chen et al (2006) to express the relationship between demand and cost, although other monotonically decreasing, continuous functions can be substituted in.

$$d_{rs} = d_{rs}^{0} \exp(-\gamma c_{rs}) \tag{6.4}$$

Where

d_{rs} is the demand from origin *r* to destination *s*, for *r*s ∈ W OD pairs;

c_{rs} is the path cost from origin *r* to destination *s*, for *r*s ∈ W;

d_{rs}^{0} is the base demand when $c_{rs} = 0$, for *r*s ∈ W;

γ is a parameter for calibrating the elasticity of average demand in the network.

Note that the inverse function is denoted with $D^{-1}(d_{rs})$. The Bureau of Public Roads (BPR) link performance function is used to relate the travel time on each link to the flow on the link, although other monotonically increasing, continuous functions with a capacity parameter can be substituted in. The function is shown in Eq. (3.10) in Chapter 3. Drivers are assumed to behave under Wardrop's principle of user equilibrium.

6.3.1. Stochastic Capacity Simulation

To model the stochastic capacity scenarios, the capacity parameter K_a from Eq. (3.10) is treated as a random variable that is uniformly distributed between $[0, \overline{K}_a]$ and conditioned on an *M*-dimensional Bernoulli variable θ. The term *failure* of a link $a \in A$ is used in this research to indicate the occurrence of $\theta_a = 1$ leading to an independent random degradation. $\overline{K}_a$ is the non-degraded capacity value and M is the number of links in the network.

$$Pr(K_a \leq x|\theta_a = 1) = \frac{x}{\overline{K}_a} \tag{6.5}$$

The method from Curtis et al (2006) is used to transform the Bernoulli random variables to standard normal variables. The orthogonal transform method from Chang et al (1994) is then used to simulate the joint distribution with uncorrelated Monte Carlo sample marginal distributions.

Because of the computational expense imposed by the number of scenarios, an efficient sampling method called Latin hypercube sampling (LHS) is used to generate the random seeds. LHS is a stratified random sampling technique that breaks down the sampling distribution into multiple regions to ensure full coverage of the range of the distribution in the most efficient manner. This sampling method is used by Chen et al (2006) because it outperforms the Monte Carlo method. In particular, the LHS Matlab utility developed by Budiman (2004) is used for convenience.

To obtain the joint distribution of the Bernoulli variable, Monte Carlo simulation is used to generate the marginal standard normal random variables for each scenario $\omega \in S$, which are then transformed using the eigenvector and diagonal eivenvalue matrix of the standard normal correlation matrix to a correlated vector of uniform random values.

$$\tilde{\theta}_a = 1 \text{ if } \mu_a \geq X_a(\omega) \tag{6.6}$$

$$X(\omega) = \Phi_{0,1}\left(V\Lambda^{1/2}\varepsilon_M(\omega)\right) \tag{6.7}$$

Where

$\varepsilon_M \in \mathbb{R}^S$ is a sample $\omega \in S$ of a standard normally distributed random variable;

V is the eigenvector of the standard normal correlation matrix ρ_0;

Λ is the diagonal matrix of eigenvalues from the standard normal correlation matrix ρ_0;

$X \in \mathbb{R}^M$ is the vector of correlated percentiles;

μ_a is the probability of degradation at link $a \in A$;

$\Phi_{0,1}$ is the cumulative distribution function of a standard normally distributed variable;

$\tilde{\theta}_a$ is the a^{th} element of the simulated *M*-dimensional Bernoulli variable.

As discussed in Chang et al (1994), there may be errors introduced in this method for estimating the joint distribution using the orthogonal transform method, but it provides a simplified scheme of accounting for dependencies between links in the scenario generation.

A comparison of the simulation results for a 76-link network is conducted for different sample sizes using the LHS method as well as a simple random sampling (SRS) method. A correlation coefficient of 0.25 is used for the same pairs of links as in the Sioux Falls network described in Appendix D. The results are shown in TABLE 6-1.

TABLE 6-1. Comparison of Capacity Failure Simulation for different *S* and sampling methods

	RMSE			
	SRS μ	**LHS μ**	**SRS ρ**	**LHS ρ**
S=50	0.0545	0.0325	0.1452	0.1427
S=100	0.0353	0.0233	0.1061	0.1051
S=200	0.0224	0.0143	0.0741	0.0741
S=1000	0.0106	0.0067	0.0401	0.0400

The root mean squared error (RMSE) of the mean values of the Bernoulli distributions (SRS μ, LHS μ) fall within 3 percent using the LHS method for a sample size of 100. The RMSE of the correlation coefficients of the Bernoulli variables (SRS ρ, LHS ρ) are approximately 10 percent for 100 samples, and decreases to 4 percent for 1000 samples. The mean values of the coefficients that should be 0.25 are summarized under (SRS Est. ρ, LHS Est. ρ).

These results favor the LHS approach over the SRS approach for computational efficiency, and verify the convergence of the simulation model towards the true probabilities as the number of simulation scenarios increases to infinity.

A qualitative comparison of incident capacity modeling methodologies is presented in TABLE 6-2, to illustrate the versatility of the proposed capacity model in handling a diverse set of incidents. The types of incidents are not meant to be comprehensive, but more illustrative in nature.

TABLE 6-2. Comparison of Capacity Models for Different Types of Incidents

Type of Incident	Proposed Capacity Modeling	Estimation/Calibration	Multivariate Normal	Independent Links	Cause-Based
Road accident	Include rubbernecking and upstream correlations, Bernoulli probabilities factored by length of link; use discrete capacity degradation for lanes instead of uniform distribution	Determine probability distributions for number of lanes affected by typical accident from historical data; use accident rates per mile to determine Bernoulli probabilities	Cannot model discrete lane degradation	No correlations, so cannot model rubbernecking and upstream impacts	Need to define generic correlations between traffic volume on link, downstream, and along opposite direction plus the independent conditional link probabilities
Natural disaster	Include geography-based correlations (e.g. link proximity to floods or earthquake faults); multiple regimes for disaster threats; use triangular distribution for degradation	Use occurrence rates of disasters for Bernoulli probabilities; define regimes based on threat of occurrence (e.g. higher hurricane warning levels result in higher Bernoulli probabilities	Can handle geography-based correlations, but can't handle low probability and high severity simultaneously	Cannot model correlations based on proximity to disaster sources	Need to enumerate all the possible occurrence scenarios by cause (flood from earthquake, flood from storm, flood from north, etc.)
Terrorist threat	Include no correlation or use criticality of links by demand; multiple regimes for security threats; degradation may need to be scenario-based	Regimes would be used to model different threat scenarios or game-theoretic models with security agencies; higher threat levels would have higher Bernoulli probabilities	Cannot handle low probability high severity situations	May be ok for some scenarios but not for large-scale coordinated attack scenarios	Should work well since the scenario planning approach would also be cause-based
Infrastructure power failure	Model link correlation based on power grid failure tree	Bernoulli probabilities should be obtained from utility agencies handling the power grid for the infrastructure	Works fine	Cannot model power grid correlations	Can assign grid failure tree to causes
Extreme weather	Bernoulli probabilities factored by length of link; multiple regimes for different seasons	Data can be collected for link capacities as a function of weather conditions for multiple seasons	Works well for continuous degradation from weather	Cannot model the observed correlations from weather due to geography and geometries	Depending on the weather condition which may involve many causes
Infrastructure deterioration	Bernoulli probabilities factored by length of link; multiple regimes based on different pavement/bridge performance categories	Data for impact of deterioration of pavement or structure on links can be obtained	Works fine	Should be ok but it would need to have continuous degradation	Causes would generally be confined to links so it would be the same as the multivariate normal
Transit rail failure	Include correlation along transit routes in a transit network	Rail reliability data can be used to determine Bernoulli probabilities	Works fine	Cannot model correlations from connected transit routes	Need to enumerate the causes for each transit route

The types of incidents are listed in the first column. The customization of the proposed capacity model for each particular incident is shown in the second column, along with the estimation/calibration issues in the third column. This is in comparison to the using multivariate normal distributions such as Chen et al (2002) for the fourth column, using independent link failures by Lo and Tung (2003) in the fifth column, and the caused-based modeling by Sumalee and Watling (2003, 2008) in the last column.

Considerations for combinations of multiple incident risks can be done, but the major challenge would be estimating the joint distribution parameters. For example, trying to combine blizzard effects with road accidents would require accident data that includes presence of snow, where the marginal distributions for snow or accidents should be the individual distributions. Copulas, as introduced earlier, might be a good mechanism for handling this with only the marginal distributions and observed data, but it is beyond the scope of this paper.

6.3.2. Multi-Objective Robust Toll Pricing Formulation

To illustrate the use of a robust approach to network toll pricing in an operational setting, a multi-objective mean-variance formulation similar to Chen et al (2006) is chosen. Eq. (6.8) is a vector of two objectives, one that maximizes the expected social welfare and one that minimizes the variance of the social welfare. The stochastic element in the proposed model is the capacity of the links in the network as opposed to the OD demand. The objective function of the toll pricing problem is also chosen to be social welfare maximization as shown in Eq. (6.9).

$$\max F = (F_1, F_2) \tag{6.8}$$

Where

$$F_1 = E[Z_1(y, \omega)]$$

$$F_2 = -V[Z_1(y, \omega)] = -E[Z_1^2(y, \omega)] + E^2[Z_1(y, \omega)]$$

F_1 and F_2 are the first and second moments, respectively, of the stochastic social welfare. The toll pricing problem is a non-convex bi-level problem where the upper level objective function is shown in Eq. (6.9) with constraints in Eq. (6.10) – (6.11), where x_a and d_{rs} are determined by the lower level problem defined in Eq. (6.12) – (6.15).

$$Z_1 = \max \sum_{rs \in W} \int_0^{d_{rs}(y)} D_{rs}^{-1}(s) ds - \sum_{a \in A} t_a\big(x_a(y), K(\omega)\big) x_a(y) \tag{6.9}$$

Subject to

$$y_{max} \geq y_a \geq 0,\ \ x_a \geq 0,\ \ a \in \bar{A} \tag{6.10}$$

$$y_a = 0,\ \ x_a \geq 0,\ \ a \in A \backslash \bar{A} \tag{6.11}$$

$$Z_2 = \min \sum_{a \in A} \int_0^{x_a} \left(t_a(v, K_a(\omega)) + \frac{1}{\psi} y_a \right) dv - \sum_{rs \in W} \int_0^{d_{rs}(y)} D_{rs}^{-1}(s) ds \quad (6.12)$$

Subject to

$$\sum_k f_k^{rs} = d_{rs}, \; rs \in W \quad (6.13)$$

$$x_a = \sum_{rs \in W} \sum_k f_k^{rs} \delta_{a,k}^{rs}, \quad \forall a \quad (6.14)$$

$$f_k^{rs} \geq 0, \quad \forall k, rs \quad (6.15)$$

Where

d_{rs} is the demand function along OD *rs*, where an exponential function is used as shown in Eq. (6.4)

x_a is the flow at link *a*

t_a is the travel time across link *a* as a function of flow, determined by Eq. (3.10)

y_a is the toll price set on link $a \in A$

A is the set of links

$\bar{A}$ is the subset of links that can be priced, where the complement is $A \backslash \bar{A}$

ψ is the value of time (assumed homogeneous)

f_k^{rs} is the flow along path *k* for O-D pair *rs* $\in W$

$\delta_{a,k}^{rs}$ is the link-path incidence, 1 if link *a* is on path *k* for OD pair *rs*, 0 otherwise

$\omega \in S$ is one realization of the capacities K_a at each link *a*, for up to *S* scenarios

Scenario simulation is used to obtain the expected value and variance of the social welfare. As shown in Sharma et al (2009), a multi-objective robust network design problem can result in a non-convex Pareto optimal frontier. As pointed out in Das and Dennis (1997), solution methods that use *a priori* articulation of preferences such as minimizing different weighted sums of the objectives only work if the Pareto frontier is convex. Therefore, heuristics are necessary to obtain an approximate Pareto optimal set.

In the multi-objective problems and methods reviewed by Marler and Arora (2004), the genetic algorithm (GA) is discussed as one of the more popular direct methods for solving multi-objective problems with *a posterior* articulation of preferences. Indeed, this is the method used by Chen et al (2006) and Sharma et al (2009). One of the faster GA methods is the NSGA II algorithm by Deb et al (2002). They compare their algorithm to other multi-objective heuristics using several simple multi-objective problems with known solutions.

However, Marler and Arora (2004) mentioned there is a high computational expense to using a genetic algorithm to solve the multi-objective problem despite its effectiveness in problems that contain both discrete and continuous decision variables. This is particularly the case for robust network optimization problems, which can feature high dimensionality and computationally costly function evaluations. The computational bottleneck would be in the function computations, not in the sorting routines. In this case, an alternative solution algorithm is proposed for its computational efficiency relative to the number of function evaluations.

6.3.3. MO-RBF Solution Algorithm for Robust Toll Pricing Problem

The solution algorithm presented at the end of Chapter 5 is shown to be a faster converging algorithm for multi-objective problems compared to the NSGA II in terms of number of function evaluations. The MO-RBF algorithm is used to solve the robust toll pricing problem within each regime, as summarized in FIGURE 6-1.

6.4 NUMERICAL TESTS ON SIOUX FALLS, SD

A numerical test is conducted with the Sioux Falls network shown in Appendix D. The following changes were made. The ten links that are investment candidates in the continuous network design problem are changed to be the toll links. The value-of-time parameter is set to $\psi = \$10/\text{hour}$. For the parameters in Eq. (6.4), $\gamma = 1$ for convenience and the base demand is set to the fixed OD demand values from the Sioux Falls network.

In first-best marginal cost toll pricing, the difference between the marginal travel time and the average travel time at the SO equilibrium represents the optimal toll. A maximum toll of $10 is used for the following tests.

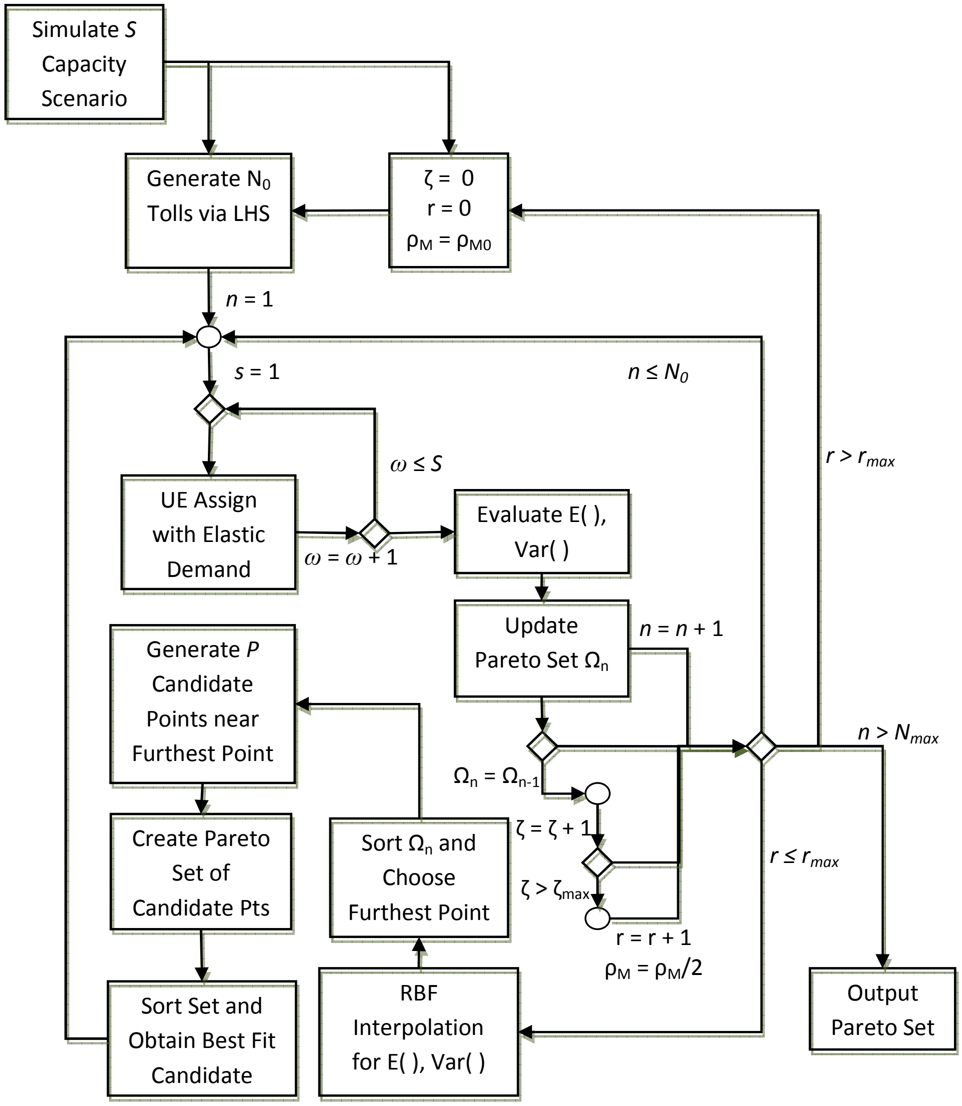

FIGURE 6-1. Multi-objective stochastic response surface algorithm, MO-RBF, for robust toll pricing.

The multi-start MSRBF parameters are set to n_0 = 21, ρ_{M0} = 0.1*Y_{max}, r_{max} = 5, and ζ_{max} = 10, P = 1000, and N_{max} = 2500. For the elastic demand user equilibrium, the Frank-Wolfe algorithm shown in Sheffi (1985) is employed to obtain the solution with a maximum of 100 iterations and tolerance of 0.01.

TABLE 6-3 provides the social welfare value for the base “No Toll, No Failures” condition as well as the “Toll with No Failures” condition. The optimal second-best tolls in the No Failures scenario is obtained using the multi-start MSRBF algorithm by Regis and Shoemaker (2007) and tested in the first part of Chapter 5.

TABLE 6-3. Sioux Falls Base Deterministic Social Welfare

	Social Welfare (kveh*hr)
No Toll UE	334.998
No Toll SO	342.208
Second-Best Toll UE	335.440

The key observation from these base values is that the deterministic social welfare bounded by the first best SO solution does not appear to be significantly higher than the UE solution, and second-best toll pricing of the 10 links in Sioux Falls appears to have negligible benefits. A network manager considering these 10 links under a deterministic setting would dismiss them as potential locations for pricing, and in fact may not even consider toll pricing to be a viable strategy at all given the limited upper bound from the system optimal solution. However, under a multiple regime volatile setting the same set of toll links can offer a much wider range of strategies in managing the network robustness.

6.4.1. Sensitivity Analysis

First, the stochastic convergence of the solution algorithm in terms of the number of scenarios is empirically tested for the Sioux Falls network with S = 250 and maximum iterations N_{max} = 200. The algorithm is applied to the 5% independent Bernoulli probability failure regime, using P = 2000 candidate points for the MO-RBF. The run time is 1927 min, compared to a run time of 3399 min for S = 300 and N_{max} = 300. The Pareto sets are shown in FIGURE 6-2. The computations were performed on Matlab 7.7.0 (R2008b) on a Windows XP Professional 2002 SP3, Intel Core2 Quad CPU with Q6600 @2.40GHz and 2GB RAM.

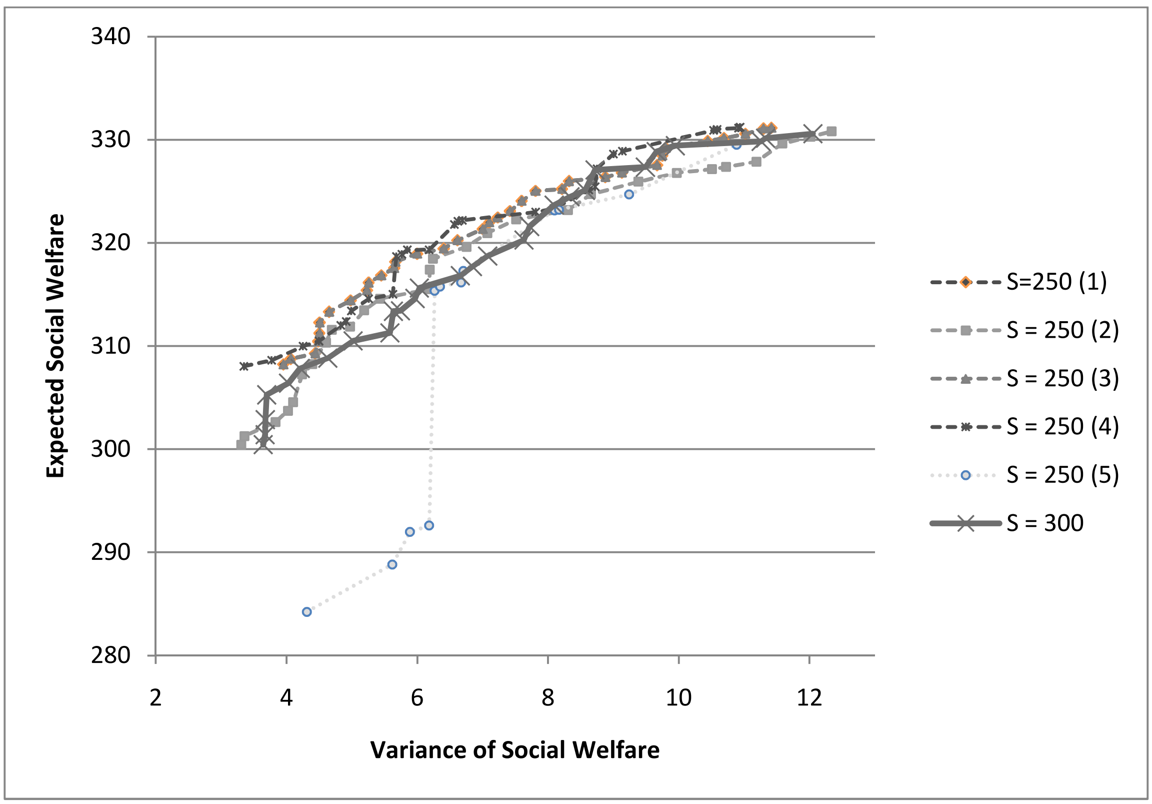

FIGURE 6-2. 5% failure regime solution space for different number of scenarios *S* and iterations N_{max}.

The ϵ-indicator is a binary performance measure defined by Zitzler et al (2003) as an effective quantitative measure to compare two approximate Pareto sets. First, an objective value $z^1 = (z_1^1, z_2^1, \dots, z_n^1) \in Z$ is defined to ϵ-dominate another objective vector $z^2 = (z_1^2, z_2^2, \dots, z_n^2) \in Z$, i.e. $z^1 \succcurlyeq_\epsilon z^2$, if and only if $\forall 1 \leq i \leq n: z_i^1 \leq \epsilon z_i^2$ for a given $\epsilon > 0$.

Then the ϵ-indicator $I_\epsilon(A, B)$ for two approximate Pareto sets *A, B* is defined as the minimum factor ϵ such that any objective vector in *B* is ϵ -dominated by at least one objective vector in *A*. In other words, if $z^1 \succ z^2$ then these exists $\epsilon < 1$ such that z^1 ϵ-dominates z^2, for $z^1 \in A$ and $z^2 \in B$.

$$I_\epsilon(A, B) = \max_{z^2 \in B} \min_{z^1 \in A} \max_{1 \leq i \leq n} \frac{z_i^1}{z_i^2} \tag{6.16}$$

One application of the ϵ-indicator is its measure of stochastic convergence for approximate Pareto sets. From intuition, as the number of scenarios of the MO-RBF approaches infinity, the variance of the ϵ-indicator of repeated Pareto set approximations with a common set should approach zero. This measure can be used as a tolerance for determining the minimum number of scenarios.

The five sample runs of S = 250 and N_{max} = 200 results in FIGURE 6-2 are compared to the S = 300 and N_{max} = 300 run to obtain the five ϵ-indicators $I_\epsilon(A^i_{S=250, N_{max}=200}, B_{S=300, N_{max}=300})$ for *i* = 1 to 5: 1.0081, 1.0133, 1.0847, 1.0055, and 1.1829. The standard error of the 5 indicators at S = 250 and N_{max} = 200 is 0.0343, which is a tolerable error. Note that runs 1 – 4 are very similar, with 1 and 3 almost identical. However, note that run number 5 starts off with a very different pattern than the others. Higher values of S and N_{max} should overcome this variation.

Comparing the S = 300 and N_{max} = 300 run to the deterministic capacity toll conditions, the maximum social welfare under 5% failure rates decreases slightly to 330.58 kveh*hrs, although the variance at that solution point is 12.05 kveh^2*hrs^2. However, the operations manager can choose a solution that would reduce the variance by 70% to 3.64 kveh^2*hrs^2 by allowing the expected social welfare to worsen by 9% to 300.44 kveh*hrs. Depending on the risk averseness of the manager in this regime, a wide range of toll prices can be set to manage the robustness of their network against link degradation.

6.4.2. Flexible Robust Toll Pricing with 5% and 50% Regimes

The transportation operations manager for Sioux Falls may be setting the toll prices to incorporate robustness against incidents that fall under two different regimes instead: a low likelihood regime with an independent failure rate of 5% at each link, and a high likelihood regime with 50% independent failure rates. For example, the two regimes may represent a dry season versus a wet season when considering roadway flooding.

The "PF75" to "PF300" represent the approximate Pareto solution set for the 50% regime as a function of the number of iterations from n = 75 to N_{max} = 300. This shows the fast convergence of the MO-RBF algorithm in terms of the number of objective function evaluations. As the number of iterations increases from 75 to 150, 200, and 250, the $I_\epsilon(z^n, z^{300})$ decreases from 1.040 to 1.014, 1.005, and 1.005. The ϵ-indicator appears to stop improving at 200 iterations.

As a flexibility tool, the robust formulation of the toll pricing under stochastic capacity allows the operations manager to adapt their toll pricing strategy to new information regarding the state of the current regime. For example, the weather station may report one day that the wet season has set in and there is a much higher likelihood for rain to impact road capacities.

The problem of measuring the value of flexibility in a multi-objective formulation is that we need to be able to quantify the difference between two approximate objective vectors. To quantify the value of flexibility, we propose using the ϵ-indicator to compare a flexible strategy against an inflexible or static strategy from Section 6.4.1 that operates under the 5% regime all the time. The differences in the ϵ-indicator can be converted to social welfare value using a fixed conversion λ to show the benefit of a flexible strategy compared to the static strategy. The value of flexibility would be the improvement of the flexible system compared to the inflexible system in the 50% regime setting shown in FIGURE 6-3.

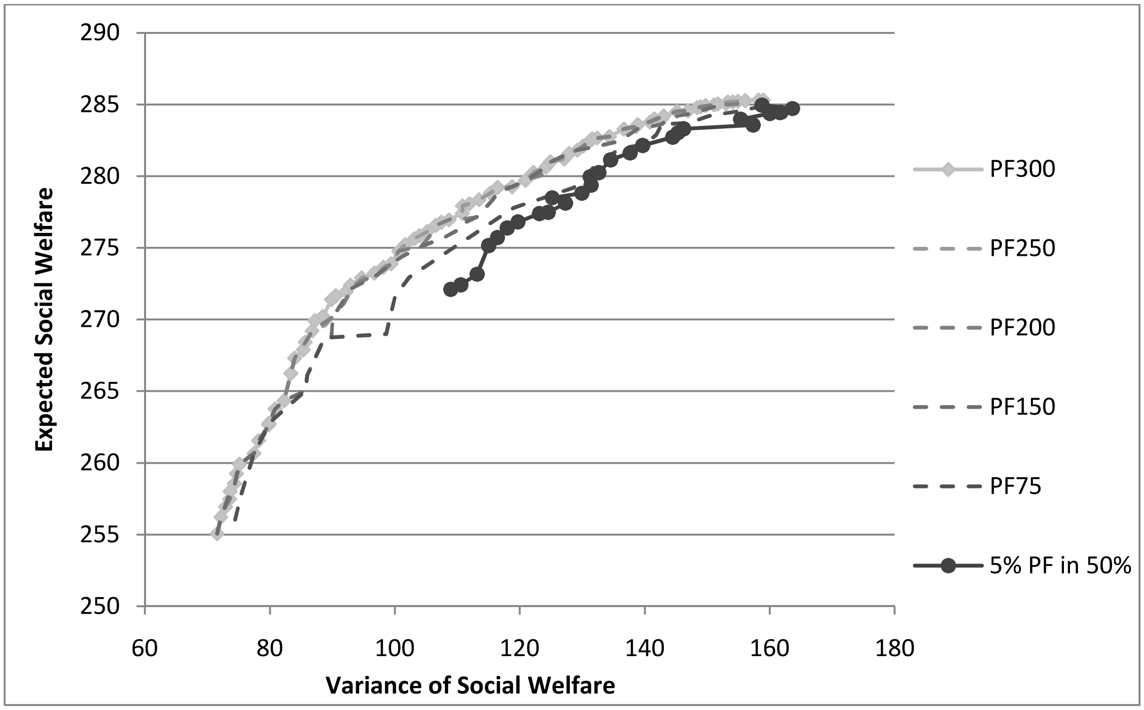

FIGURE 6-3. Pareto frontier with 5% and 50% failure regimes.

The "5% PF in 50%" shows the performance of the same solution vector for the 5% regime from FIGURE 6-2 when operating in the 50% regime. The 5% Pareto solution operating in the 50% failure regime is clearly dominated by the 50% Pareto solution.

The maximum expected value in the 50% regime further decreases from 330.58 kveh*hrs in the 5% failure regime to 285.32 kveh*hrs. Furthermore, the minimum variance increases from 12.05 $kveh^2$*hrs^2 in the 5% regime to 71.58 $kveh^2$*hrs^2. The 5% Pareto solution, however, has a maximum expected value of 284.95 kveh*hrs and minimum variance of 108.96 $kveh^2$*hrs^2 in the 50% regime.

Since the expected value objective is a maximization function, the reciprocal of ϵ is used for that objective. Hence, $I_\epsilon(5\%, 50\%) = 1.522$. The value of flexibility F can be interpreted as follows.

$$F = \lambda(I_\epsilon(A, B) - 1)p \tag{6.17}$$

Where *p* is the average proportion of time that the network falls under the 50% regime. Thus having a flexible toll system in this example with two regimes at 5% failure and 50% failure as opposed to a single regime robust toll system increases the value of the system by 0.522λ*p*, which may be in units representing a combination of expected social welfare and risk of loss of social welfare.

6.5 DISCUSSION

The goal of the research is to establish robust optimization as a tool for increasing flexibility in traffic operations rather than just in long term planning as suggested by most of the applications in the literature. Toll pricing is chosen to exemplify this approach. Second best pricing using an arbitrary set of ten links in Sioux Falls under a deterministic setting suggests negligible benefits, as shown in TABLE 6-3. However, robust toll pricing with the same set of links using a mean-variance formulation under multiple uncertainty regimes can give a decision-maker significant leverage in their flexibility. Not only will the operations managers be able to choose a set of tolls based on their personal or institutional risk averseness, as indicated in FIGURE 6-2, they can also adapt their Pareto optimal toll price sets to new uncertainty regimes as shown in FIGURE 6-3.

The stochastic capacities are modeled as a multivariate Bernoulli random variable for occurrences of degradation. Scenarios are generated by transforming the multivariate Bernoulli variables into equivalent multivariate normal distributions using the method by Curtis, which are then simulated with an orthogonal transform approach. This simulation approach is compared to existing methods as a simple approach that can nonetheless capture multiple correlations between different links for evaluating degradations in a network.

The MO-RBF algorithm proposed in Chapter 5 is used to solve the robust toll pricing problem in each regime. Numerical tests with Sioux Falls shows convergence can be achieved by the MO-RBF algorithm within 200 iterations in terms of the change in the ϵ-indicator.

The value of flexibility to modify the toll pricing to suit a particular regime is illustrated for Sioux Falls under a low occurrence season of 5% independent link failure likelihood and a high season of 50% failure likelihood. In that example, the flexibility increases the value of system in terms of the ϵ-indicator by $0.522\lambda p$, where λ is a fixed conversion rate of the ϵ to some value, and p is the proportion of time under the 50% regime setting.

"Remember not only to say the right thing in the right place, but far more difficult still, to leave unsaid the wrong thing at the tempting moment." – Benjamin Franklin

CHAPTER 7 CONCLUSION

The goals of this research were to explore various models, algorithms, and policies related to flexible transportation network management under uncertainty. To summarize the motivation of this work, let us re-state the challenge in Chapter 1 put forth by a well-regarded transportation practitioner: "*The inherited culture of today's transportation agencies is dominated by facility development and preservation. Changes are required if state and local agencies are to have a significant impact on the characteristic 21st century mobility problems of congestion, unreliability, and insecurity*".

While the solutions proposed in this research seek to remedy many of these inadequacies in the public sector in infrastructure in dealing with today's volatile environment, they are developed under a generalized flexible transportation network management that can be applied to many other related areas.

From a most general perspective, this research has 1) established a relationship between a tool for managing networks with both stationary and non-stationary uncertainty; 2) proven empirically that such network management tools can benefit from incorporating information from monitoring uncertainty; 3) improved upon state-of-the-art methods to solving problems with multiple objectives given certain criteria; 4) developed a new method for simulating scenarios with inter-related components; and 5) demonstrated how to quantify the value of flexibility for networks with stationary and non-stationary uncertainty, using network design models that may have a single or multiple objectives.

7.1 SUMMARY OF CONTRIBUTIONS

To further summarize the contributions in this research, consider the following findings categorized by policies, models, and algorithms.

7.1.1. Policies

Several policies are examined in this research. First, the traditional approach in transportation planning is questioned by using real options to show that there is a cost to committing to a preferred alternative. Depending on the volatility estimated for OD demand and congestion settings in a network, it may be more worth it to maintain a set of mutually exclusive conditional alternatives instead.

Second, a network design can gain significant flexibility by being decoupled into its individual link components. While the trade-off in such a policy is the loss of flexibility to re-design the network, the gain from being able to stage the components can be substantial.

Third, a severity threshold has been identified for which fire planning authorities can determine whether to apply more simplistic static location resource allocation models or to use more sophisticated relocation models in their day-to-day operational planning.

Fourth, incorporating non-stationary time series data directly into network models such as server relocation can improve performance of those models because they account for hysteresis. The abundance of monitored time series data in other fields such as traffic incident management, airline operations, and supply chain management can benefit significantly from this finding.

Fifth, toll pricing is one type of network design strategy that can be used to manage network robustness in a flexible manner because of its negligible switching costs. Other network design strategies that have negligible switching costs should also exhibit this property –ramp metering, signal control on arterials, project scheduling, among others.

7.1.2. Models

The network investment deferral option (NIDO) model quantifies the value of flexibility to defer and re-design a network over time. This allows decision-makers to evaluate flexible network designs in a non-stationary stochastic setting, even if they cannot solve such a design problem.

By fixing the network design at the initial time, it becomes possible to solve the optimal design with respect to the option value of the network investment. This proposed model is the network option design problem (NODP). The NODP fits especially well in the existing transportation planning practice because it can be used to determine the optimal design and decision to defer simultaneously.

By allowing the components of a network design to be invested or deferred separately, a link investment deferral option set (LIDOS) model is defined. By further constraining this model to prevent the order of link investments to change in the future, a solvable ordered link investment deferral option set (OLIDOS) model is determined. This model determines the optimal order and deferral decisions for each link or group of links in a pre-defined network design, which serves as an effective strategy to mitigate the downside risk from non-stationary uncertainty.

The k-facility p-median problem (KPMP) considers an alternative approach to modeling the p-median problem with co-location and multiple server constraints by using integers to represent the number of servers at a node. The model has applicability in areas where the number of servers relative to the number of nodes is large and the demand typically requires more than one server, such as wildfires and air tanker initial attacks. Other applications of this model may exist in the airline industry and in humanitarian logistics, where pre-positioning is on a global scale with potentially large-scale disasters.

The chance-constrained dynamic *k*-server relocation problem (CDKRP) directly incorporates hysteresis through the use of non-stationary variables as chance constraints. As discussed in Section 7.1.1, this model is more cost effective than relocation models that do not account for hysteresis.

The network degradation simulation model can simulate stationary scenarios for correlated failure occurrences between multiple links. It uses a multivariate Bernoulli random variable and finds an equivalent multivariate normal distribution representation that can then be simulated using orthogonal transforms. This method can have significant impact on the way scenario planning in network settings is handled because it provides a simple mechanism of estimating probabilities of failure with the Bernoulli variables while handling correlations between link failure occurrences.

7.1.3. Algorithms

Although the multi-option LSM algorithm is limited in its applicability, it can nonetheless be used in an algorithm to obtain the solution to the OLIDOS and thus a lower bound to the ILIDOS. The key is in recognizing that the feasible solutions to OLIDOS can be enumerated; for each enumeration the value of the link option set can be solved using one of the stylized conditions of the multi-option LSM.

The multi-objective radial basis function (MO-RBF) optimization method is shown to be a much faster heuristic than the genetic algorithm for multi-objective problems. While the algorithm is demonstrated on a simple non-convex function in Chapter 5 and on a robust toll pricing problem in Chapter 6, it can clearly be applied to any number of large-scale multi-objective problems with continuous decision variables.

7.2 FUTURE RESEARCH

Overall the direction of this research lies in practical applications of the models to real large-scale network case studies; expanding the network-based real option models into a suite of models for expanding a decision-makers network management strategies given non-stationary uncertainty; and tackling the other element of dynamic strategic planning: the decision-makers. Detailed extensions are discussed below.

7.2.1. Network-based Real Option Models

The applications and extensions to this research are abundant. Practical applications using time series OD data can provide case studies to practitioners on the benefits of these approaches to transportation planning as well as for more generalized network design investments. Other real option strategies, network design controls, and stochastic simulation models can be considered in these case studies. For example, intercity truck OD time series data is available in some regions such as Iran. This data can be used to estimate OD demand and provide dynamic strategies for investing and managing the road network for truck flows.

Specific issues also need to be addressed. The stochastic variable such as demand is treated as an exogenous variable in these models, so that changes in the network would not have an effect on the demand. The NODP solution method is a heuristic that only works with continuous design variables. The OLIDOS has computational cost issues when dealing with large numbers of projects or link combinations which will require meta-heuristics to approximate.

By looking at multi-objective network design problems under non-stationary uncertainty, it would be possible to examine how the Pareto sets of the option value vectors may hold different properties from the corresponding objective vectors.

Multiple agency collaboration and competition in network design can also be examined using real options; it can be used to determine whether an agency should join with another agency in bidding for project funding; the real option valuation of multi-agency network design game may result in different Nash equilibria from an evaluation using expected total travel costs.

7.2.2. Mobile Server Relocation Models

Future steps would involve agency collaboration to gather more data to present a realistic model for statewide implement. This includes more CDF units, more fire weather station input, more types of resources in addition to air tankers, and calibrating the parameters for resource demand and relating relocation costs to deployment time.

Future modeling research should handle different types of resources, congested links for ground resources, and some consideration for demand correlations. Other fire weather data and stochastic processes could be tested for better goodness-of-fit.

This research can also be generalized for many other network models that utilize demand under uncertainty. For example, incident management on road networks can benefit by characterizing traffic volumes with accident rates and using that information as a Markov model for multi-period relocation of toll vehicles. In supply chain networks, the location of distribution centers can benefit from the characterization of consumer demand as stochastic processes. In humanitarian logistics, the pre-positioning of resources for disaster relief may benefit from the incorporation of hysteresis in resource relocation models.

7.2.3. Global Heuristics with Radial Basis Functions

Future efforts in this area should include more rigorous tweaking and testing of the algorithms for large-scale networks. The performance of the algorithm can be empirically tested separately for the number of allowable link investments and the size of the network.

In addition, the algorithms could be refined to take advantage of the underlying network data structure or combined with other heuristics for the CNDP. For example, it is also possible to combine the RBF interpolation within an evolutionary structure such as the GA (see ESGRBF from Shoemaker et al (2007)).

7.2.4. Network Degradation under Multiple Failure Regimes

Future work in this direction should consider other methods of modeling the uncertainty, perhaps correlating the Bernoulli occurrences with the amount of degradation using copulas.

The proposed network degradation simulation model can be customized for particular real world uncertainties to quantitatively compare the model's ability to handle the wide range of failure types suggested in TABLE 6-2.

Markov processes can also be considered for evaluating correlations between different regimes. This extension would allow us to consider non-stationary stochastic behavior in regimes where the stochastic capacity distributions in a given regime is conditional on the likelihood of that regime occurring. Certain types of failure or degradation with recurrent elements, such as pavement deterioration, would find use for this.

APPENDIX A. EXAMPLE OF MULTI-OPTION INVESTMENT

The following example is from Trigeorgis (1991). An investment project is determined to have an expected profit of 1000 units if invested immediately and 1015 units cost. Using traditional NPV analysis, the project would be rejected because the NPV is negative:

$$NPV(no\ options) = V - I = 1000 - 1015 = -15$$

However, this does not account for the flexibility to adapt the decisions using a number of real options. The following list of options are available to the investor, where their premiums are obtained by standard option pricing for each one in isolation. FIGURE A-1 illustrates the different options and how they interact with each other.

option to defer investment: 147

option to abandon during construction: 34

option to contract project scale: 62

option to expand production: 133

option to switch use (abandon for salvage): 121

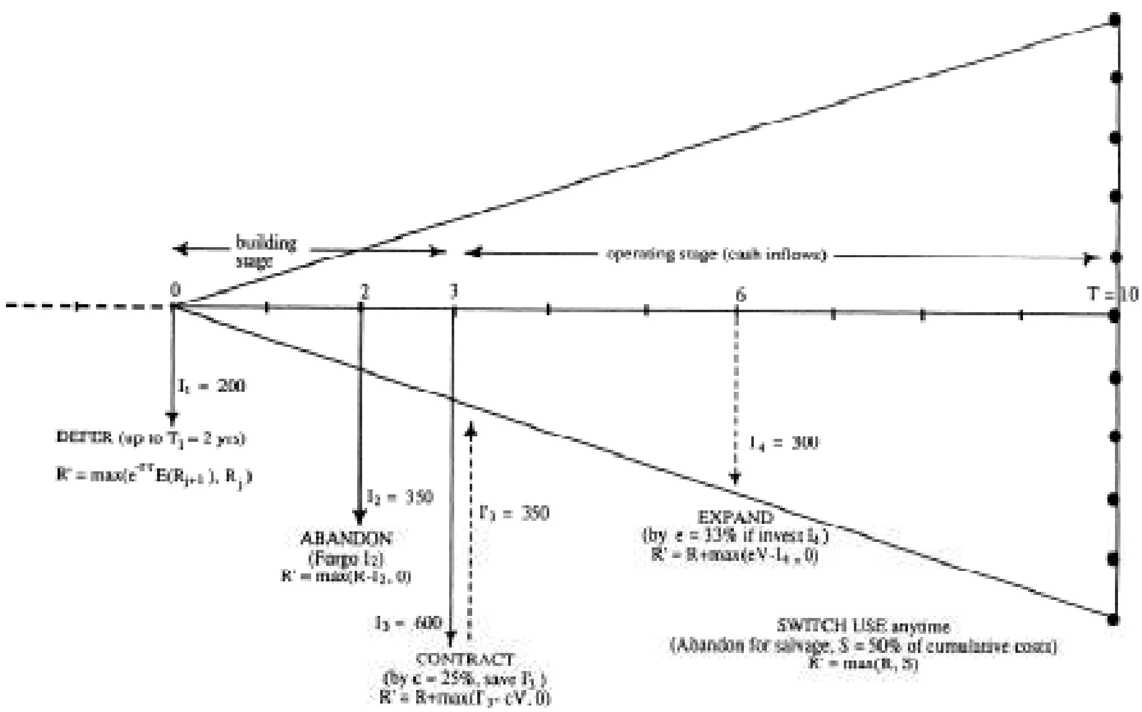

FIGURE A-1 Example complex research and development project (Trigeorgis, 1991)

Adding the individual option premiums together would result in an overestimated value of 497 units (or 482 after including the static NPV of -15 units). This method of summing the individual options up is invalid because exercising certain prior real options may kill or alter the value of subsequent options. If these interactions are taken into account by computing the options together, the combined premium value of all five options turns out to be 349, leading to an option value of 334 units. It can be observed that this amount is just 70 percent of the sum.

APPENDIX B. PROOF OF CONVERGENCE FOR LEAST SQUARES MONTE CARLO SIMULATION

The following formal definitions and proof of convergence for the LSM algorithm is from Longstaff and Schwartz (2001).

Let the probability space be defined by $(\Omega, \mathcal{F}, \Psi)$ and finite time horizon [0,*T*] where the state space Ω is the set of all possible realizations of the stochastic process between time 0 and *T* and has typical element ω representing a sample path, $\mathcal{F}$ is the sigma field of distinguishable events at time *T*, and ψ is a probability measure defined on the elements of $\mathcal{F}$. $\mathcal{F} = \{\mathcal{F}_t; \tau \in [0, T]\}$ is defined to be the augmented filtration generated by the relevant price processes for the investment, and assume that $\mathcal{F}_T = \mathcal{F}$.

At time t_2, the LSM stopping strategy is the same as the optimal strategy; the option is exercised if it is in the money. Under the given assumptions, the conditional expectation function $\Phi(\omega; t_1)$ is a function only of X_{t_1}. If $\Phi(\omega; t_1)$ satisfies the indicated conditions, then Theorem IV.9.1 of Sansone (1959) implies that the convergence of $\Phi_\Pi(\omega; t_1)$ to $\Phi(\omega; t_1)$ is uniform in Π on the set $(0,\infty)$, where the first Π Laguerre polynomials are used as the set of basis functions. This implies that for a given ϵ, there exists an Π such that $\sup_{X_{t_1}} |\Phi(\omega; t_1) - \Phi_\Pi(\omega; t_1)| < \epsilon/2$. From the integrability conditions and Theorem 3.5 of White (1984), the fitted value of the LSM regression $\widehat{\Phi}_\Pi(\omega; t_1)$ converges in probability to $\Phi_\Pi(\omega; t_1)$ as $P \to \infty$,

$$\lim_{P\to\infty} \Pr\left[\left|\Phi_\Pi(\omega; t_1) - \widehat{\Phi}_\Pi(\omega; t_1)\right| > \frac{\quad}{2}\right] = 0$$

Thus, for any ϵ, there is an Π such that

$$\lim_{P\to\infty} \Pr\left[\left|\Phi(\omega; t_1) - \widehat{\Phi}_\Pi(\omega; t_1)\right| > \quad\right] = 0$$

To complete the proof, let's partition the state space Ω into five sets:

1) The set of paths where the option is exercised at time t_1 under both the optimal and the LSM strategy;
2) The set of paths where the option is not exercised at time t_1 under either the optimal or LSM strategies;
3) The set of paths where the option is exercised at time t_1 under the LSM strategy but not under the optimal strategy;

4) The set of paths where the option is exercised at time t_1 under the optimal strategy, but not under the LSM strategy;

5) A zero-probability set of paths for which the difference between $\Phi(\omega; t_1)$ and $\widehat{\Phi}_{\Pi}(\omega; t_1)$ is greater than ϵ as $P \rightarrow \infty$.

Now consider a portfolio consisting of a long position in an option exercised using the LSM strategy, an investment of ϵ in a money market account, and a short position in an option exercised using the optimal strategy. It can be shown that cash flows are all non-negative for each path in sets 1) – 4). Since the pathwise cash flows are non-negative, averages over paths are non-negative, and the result follows from a standard no-arbitrage argument, the definition of $\pi(X)$, and the law of large numbers.

APPENDIX C. PROOF OF CONVERGENCE FOR STOCHASTIC RESPONSE SURFACE METHOD

The following formal definitions and proof of convergence for the SRS framework is from Regis and Shoemaker (2007).

First let's define two conditions required by the SRS method.

Condition C.1 – *For each* $n \geq n_0, Y_{n,1}, Y_{n,2}, \ldots, Y_{n,t}$ *are conditionally independent given the random vectors in* $\mathcal{E}_{n-1}$.

Where *n* is an iteration in the algorithm, n_0 is the initial set of iterations, Y_n is a set of randomly generated samples, X_n is the chosen candidate from the set of Y_n, *t* is the number of random candidate points, and $\mathcal{E}_n$ is the set of random samples Y_n and the chosen candidate X_n, , and $\mathcal{E}_n := \{X_1, \dots, X_{n_0}, Y_{n_0,1}, \dots, Y_{n_0,t}, \dots, Y_{n,1}, \dots, Y_{n,t}\}$.

Condition C.2 – *For any j = 1, …, t, $x \in D$ and $\delta > 0$, there exists $\nu_j(x,\delta) > 0$ such that*

$$\Pr[Y_{n,j} \in B(x,\delta) \cap D | \sigma(\mathcal{E}_{n-1})] \geq \nu_j(x,\delta)$$

for all $n \geq n_0$. Here B(x,δ) is the open ball of radius δ centered at x and $\sigma(\mathcal{E}_{n-1})$ is the σ-field generated by the random vectors in $\mathcal{E}_{n-1}$.

The following theorem states the convergence given the two conditions C.1 and C.2.

Theorem C.1 – *Let f be a function defined on $D \subseteq \mathbb{R}^d$ and suppose that x^* is the unique global minimize of f on D in the sense that $f(x^*) = \inf_{x \in D} f(x) > -\infty$ and $\inf_{x \in D, \|x - x^*\| \geq \eta} f(x) > f(x^*)$ for all $\eta > 0$. Suppose further that the SRS method generates the random vectors $\{X_n\}_{n\geq 1}$ and $\{Y_{n,1}, \dots, Y_{n,t}\}_{n \geq n_0}$ satisfying Conditions C.1 and C.2. Define the sequence of random vectors $\{X_n^*\}_{n\geq 1}$ as follows: $X_n^* = X_{n-1}^*$ otherwise. Then $X_n^* \to x^*$ almost surely.*

Proof – Fix $\epsilon > 0$ and $n \geq n_0 + 1$. Then $[X_n \in D: f(X_n) < f(x^*) + \epsilon] = [X_n \in D: |f(X_n) - f(x^*)| < \epsilon]$. Since f is continuous on x^*, there exists $\delta(\epsilon) > 0$ such that $|f(x) - f(x^*)| < \epsilon$ whenever $\|x - x^*\| < \delta(\epsilon)$. Hence, $[X_n \in D: |f(X_n) - f(x^*)| < \epsilon] \supseteq [X_n \in D: \|X_n - x^*\| < \delta(\epsilon)]$, and so,

$$\Pr[X_n \in D: |f(X_n) - f(x^*)| < \epsilon | \sigma(\mathcal{E}_{n-2})] \geq \Pr[X_n \in D: \|X_n - x^*\| < \delta(\epsilon) | \sigma(\mathcal{E}_{n-2})]$$
$$= \Pr\left[X_n \in B\left(x^*, \delta(\epsilon)\right) \cap D | \sigma(\mathcal{E}_{n-2})\right]$$

Observe that if $\left(\mathrm{Y}_{\mathrm{n-1,j}}\right)_D \in B\left(x^*, \delta(\epsilon)\right) \cap D$ for each j = 1, …, t, then the evaluation point $X_n \in B\left(x^*, \delta(\epsilon)\right) \cap D$. Hence,

$$\Pr\left[X_n \in B\left(x^*, \delta(\epsilon)\right) \cap D | \sigma(\mathcal{E}_{n-2})\right]$$
$$\geq \Pr\left[\left(Y_{n-1,j}\right)_D \in B\left(x^*, \delta(\epsilon)\right) \cap D, j = 1, \dots, t | \sigma(\mathcal{E}_{n-2})\right]$$
$$\geq \Pr\left[Y_{n-1,j} \in B\left(x^*, \delta(\epsilon)\right) \cap D, j = 1, \dots, t | \sigma(\mathcal{E}_{n-2})\right]$$
$$= \prod_{j=1}^{t} \Pr\left[Y_{n-1,j} \in B\left(x^*, \delta(\epsilon)\right) \cap D | \sigma(\mathcal{E}_{n-2})\right]$$
$$\geq \prod_{j=1}^{t} \nu_j\left(x^*, \delta(\epsilon)\right) =: L(\epsilon) > 0$$

Where the equality and inequality involving the product sign follow from conditions C.1 and C.2. Thus, the two equations above lead to:

$$\Pr[X_n \in D: f(X_n) < f(x^*) + \epsilon | \sigma(\mathcal{E}_{n-2})] \geq L(\epsilon)$$

By following the same argument as in the proof of the theorem in p.40 of Spall (2003), we obtain $X_n^* \to x^*$ almost surely.

APPENDIX D. SIOUX FALLS NETWORK PARAMETERS

The Sioux Falls, SD network is commonly used throughout the dissertation in a number of chapters because of its convenience as a small network for testing traffic assignment and network design models. While many variations exist (see Bar Gera, 2009) since it was first used by LeBlanc (1975), the parameters used for testing in the chapters are based on FIGURE D-1 and FIGURE D-2.

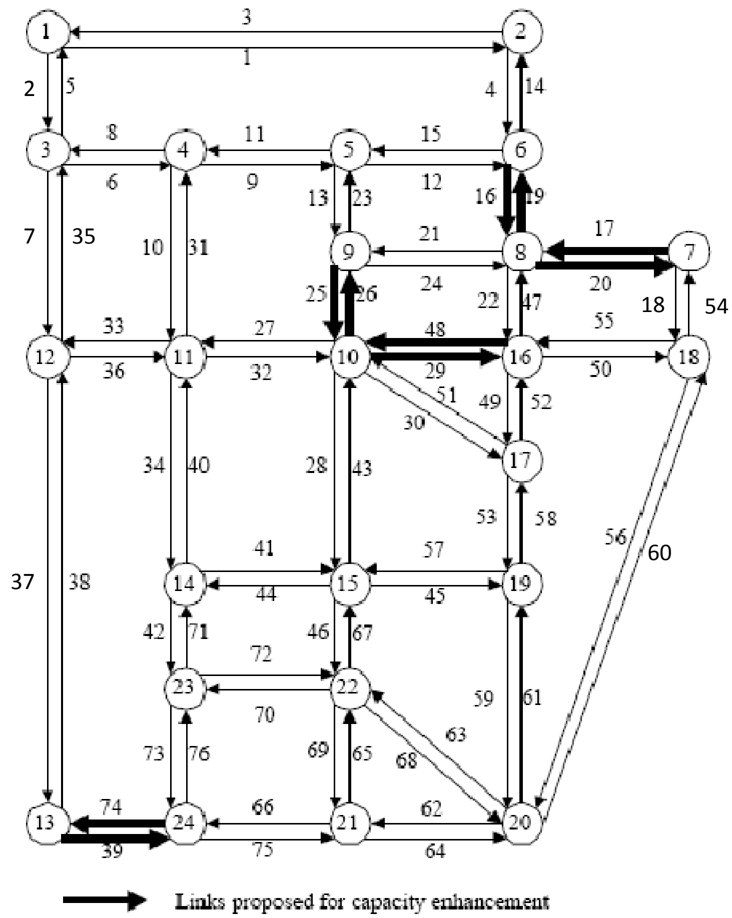

FIGURE D-1. Test Network – Sioux Falls, SD (Chen and Yang, 2004).

$$c_a(f_a, y_a) = A_a - B_a(f_a/(K_a + y_a))^4$$

$$Z(y) = \sum_a (c_a(f_a, y_a) \cdot f_a + 0.001\, d_a \quad y_a^2)$$

Arcs	A_a (hours)	B_a (hours)	K_a (thousand vehicles)	d_a (thousand dollars)
1 and 3	0.06	0.0090	25.9002	
2 and 5	0.04	0.0060	23.4035	
4 and 14	0.05	0.0075	4.9582	
6 and 8	0.04	0.0060	17.1105	
7 and 35	0.04	0.0060	23.4035	
9 and 11	0.02	0.0030	17.7828	
10 and 31	0.06	0.0090	4.9088	
12 and 15	0.04	0.0060	4.9480	
13 and 23	0.05	0.0075	10.0000	
16 and 19	0.02	0.0030	4.8986	26.00
17 and 20	0.03	0.0045	7.8418	40.00
18 and 54	0.02	0.0030	23.4035	
21 and 24	0.10	0.0150	5.0502	
22 and 47	0.05	0.0075	5.0458	
25 and 26	0.03	0.0045	13.9158	25.00
27 and 32	0.05	0.0075	10.0000	
28 and 43	0.06	0.0090	13.5120	
29 and 48	0.05	0.0075	5.1335	48.00
30 and 51	0.08	0.0120	4.9935	
33 and 36	0.06	0.0090	4.9088	
34 and 40	0.04	0.0060	4.8765	
37 and 38	0.03	0.0045	25.9002	
39 and 74	0.04	0.0060	5.0913	34.00
41 and 44	0.05	0.0075	5.1275	
42 and 71	0.04	0.0060	4.9248	
45 and 57	0.04	0.0060	15.6508	
46 and 67	0.04	0.0060	10.3150	
49 and 52	0.02	0.0030	5.2299	
50 and 55	0.03	0.0045	19.6799	
53 and 58	0.02	0.0030	4.8240	
56 and 60	0.04	0.0060	23.4035	
59 and 61	0.04	0.0060	5.0026	
62 and 64	0.06	0.0090	5.0599	
63 and 68	0.05	0.0075	5.0757	
65 and 69	0.02	0.0030	5.2299	
66 and 75	0.03	0.0045	4.8854	
70 and 72	0.04	0.0060	5.0000	
73 and 76	0.02	0.0030	5.0785	

FIGURE D-2 Sioux Falls Link Parameters (Suwansirikul et al, 1987).

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1		0.11	0.11	0.55	0.22	0.33	0.55	0.88	0.55	1.43	0.55	0.22	0.55	0.33	0.55	0.55	0.44	0.11	0.33	0.33	0.11	0.44	0.33	0.11
2	0.11		0.11	0.22	0.11	0.44	0.22	0.44	0.22	0.66	0.22	0.11	0.33	0.11	0.11	0.44	0.22	0.00	0.11	0.11	0.00	0.11	0.00	0.00
3	0.11	0.11		0.22	0.11	0.33	0.11	0.22	0.11	0.33	0.33	0.22	0.11	0.11	0.11	0.22	0.11	0.00	0.00	0.00	0.00	0.11	0.11	0.00
4	0.55	0.22	0.22		0.55	0.44	0.44	0.77	0.77	1.32	1.54	0.66	0.66	0.55	0.55	0.88	0.55	0.11	0.22	0.33	0.22	0.44	0.55	0.22
5	0.22	0.11	0.11	0.55		0.22	0.22	0.55	0.88	1.10	0.55	0.22	0.22	0.11	0.22	0.55	0.22	0.00	0.11	0.11	0.11	0.22	0.11	0.00
6	0.33	0.44	0.33	0.44	0.22		0.44	0.88	0.44	0.88	0.44	0.22	0.22	0.11	0.22	0.99	0.55	0.11	0.22	0.33	0.11	0.22	0.11	0.11
7	0.55	0.22	0.11	0.44	0.22	0.44		1.10	0.66	2.09	0.55	0.77	0.44	0.22	0.55	1.54	1.10	0.22	0.44	0.55	0.22	0.55	0.22	0.11
8	0.88	0.44	0.22	0.77	0.55	0.88	1.10		0.88	1.76	0.88	0.66	0.66	0.44	0.66	2.42	1.54	0.33	0.77	0.99	0.44	0.55	0.33	0.22
9	0.55	0.22	0.11	0.77	0.88	0.44	0.66	0.88		3.08	1.54	0.66	0.66	0.66	0.99	1.54	0.99	0.22	0.44	0.66	0.33	0.77	0.55	0.22
10	1.43	0.66	0.33	1.32	1.10	0.88	2.09	1.76	3.08		4.40	2.20	2.09	2.31	4.40	4.84	4.29	0.77	1.98	2.75	1.32	2.86	1.98	0.88
11	0.55	0.22	0.33	1.65	0.55	0.44	0.55	0.88	1.54	4.29		1.54	1.10	1.76	1.54	1.54	1.10	0.11	0.44	0.66	0.44	1.21	1.43	0.66
12	0.22	0.11	0.22	0.66	0.22	0.22	0.77	0.66	0.66	2.20	1.54		1.43	0.77	0.77	0.77	0.66	0.22	0.33	0.44	0.33	0.77	0.77	0.55
13	0.55	0.33	0.11	0.66	0.22	0.22	0.44	0.66	0.66	2.09	1.10	1.43		0.66	0.77	0.66	0.55	0.11	0.33	0.66	0.66	1.43	0.88	0.88
14	0.33	0.11	0.11	0.55	0.11	0.11	0.22	0.44	0.66	2.31	1.76	0.77	0.66		1.43	0.77	0.77	0.11	0.33	0.55	0.44	1.32	1.21	0.44
15	0.55	0.11	0.11	0.55	0.22	0.22	0.55	0.66	1.10	4.40	1.54	0.77	0.77	1.43		1.32	1.65	0.22	0.88	1.21	0.88	2.86	1.10	0.44
16	0.55	0.44	0.22	0.88	0.55	0.99	1.54	2.42	1.64	4.84	1.54	0.77	0.66	0.77	1.32		3.08	0.55	1.43	1.76	0.66	1.32	0.55	0.33
17	0.44	0.22	0.11	0.55	0.22	0.55	1.10	1.54	0.99	4.29	1.10	0.66	0.55	0.77	1.65	3.08		0.66	1.87	1.87	0.66	1.87	0.66	0.33
18	0.11	0.00	0.00	0.11	0.00	0.11	0.22	0.33	0.22	0.77	0.22	0.22	0.11	0.11	0.22	0.55	0.66		0.33	0.44	0.11	0.33	0.11	0.00
19	0.33	0.11	0.00	0.22	0.11	0.22	0.44	0.77	0.44	1.98	0.44	0.33	0.33	0.33	0.88	1.43	1.87	0.33		1.32	0.44	1.32	0.33	0.11
20	0.33	0.11	0.00	0.33	0.11	0.33	0.55	0.99	0.66	2.75	0.66	0.55	0.66	0.55	1.21	1.76	1.87	0.44	1.32		1.32	2.64	0.77	0.44
21	0.11	0.00	0.00	0.22	0.11	0.11	0.22	0.44	0.33	1.32	0.44	0.33	0.66	0.44	0.88	0.66	0.66	0.11	0.44	1.32		1.98	0.77	0.55
22	0.44	0.11	0.11	0.44	0.22	0.22	0.55	0.55	0.77	2.86	1.21	0.77	1.43	1.32	2.86	1.32	1.87	0.33	1.32	2.64	1.98		2.31	1.21
23	0.33	0.00	0.11	0.55	0.11	0.11	0.22	0.33	0.55	1.98	1.43	0.77	0.88	1.21	1.10	0.55	0.66	0.11	0.33	0.77	0.77	2.31		0.77
24	0.11	0.00	0.00	0.22	0.00	0.11	0.11	0.22	0.22	0.88	0.66	0.55	0.77	0.44	0.44	0.33	0.33	0.00	0.11	0.44	0.55	1.21	0.77	

FIGURE D-3 Sioux Falls Travel Demand Matrix (Suwansirikul et al, 1987).

To be consistent with numerical tests from the literature, the budget is set equal to 5500 (in thousands of U.S. dollars), no upper bounds for y_a are used (besides the one implied by the budget constraint for a single link), and only links 16, 17, 19, 20, 25, 26, 29, 39, 48, and 74 are considered for expansion. The construction cost function from eq. (3.4) is assumed to be a quadratic function (γ = 2). In the literature, a rate of 0.001 is used to convert the budget to travel time savings, so that 5500 is equivalent to 5.5 VHT.

APPENDIX E. CONIDO TEST RESULTS

The CONIDO solution using LSM for a range of *Π* basis functions is shown below.

TABLE E-1. Thirty Option Value ($M) Runs, P = 30 Sample Paths

	CONIDO Value				Fixed Design Deferral Value			
#	*Π* = 3	*Π* = 4	*Π* = 5	*Π* = 6	*Π* = 3	*Π* = 4	*Π* = 5	*Π* = 6
1	23.53	28.25	30.67	31.16	23.00	25.77	27.21	29.23
2	20.43	20.6	24.7	24.87	19.94	19.85	22.77	22.54
3	19.73	20.66	23.32	21.94	19.73	20.72	22.63	22.85
4	25.93	35.32	38.7	40.58	24.46	31.34	36.62	37.43
5	20.19	20.8	20.18	21.45	20.21	19.73	19.87	20.45
6	22.36	23.13	23.64	25.04	21.33	22.18	22.52	23.66
7	25.32	26.13	29.43	30.79	25.39	25.33	27.52	28.43
8	20.56	20.97	22.26	22.8	20.13	20.37	20.97	22.06
9	21.05	24.06	23.24	25.25	20.34	22.95	22.35	24.04
10	20.88	20.98	20.76	21.62	20.6	20.49	21.05	20.98
11	20.5	21.95	21.82	21.64	19.73	20.62	20.78	20.11
12	19.56	21.53	22.5	22.51	20.21	20.46	21.68	22.27
13	19.73	21.07	21.31	22.04	19.73	19.73	20.44	20.29
14	24.36	26.89	26.41	26.61	22.93	24.37	24.74	26.36
15	23.63	25.37	26.1	27.83	22.46	23.43	24.38	25.65
16	25.94	26.24	27.94	31.15	24.56	24.28	26.26	28.37
17	20.03	19.73	21.1	20.78	19.84	19.76	19.73	20.16
18	22.95	23.23	24.87	25.63	22.03	23.10	24.06	25.32
19	19.83	19.73	20.08	19.73	19.73	19.73	19.99	19.73
20	20.82	21.4	22.34	23.71	20.77	21.07	21.63	22.73
21	23.26	24.69	23.98	25.75	22.30	23.49	23.29	24.12
22	20.21	19.75	22.65	23.41	19.73	19.73	22.02	21.12
23	21.02	22.07	22.15	23.21	20.55	20.74	21.49	22.45
24	20.38	22.13	22.5	22.68	20.11	21.58	22.41	22.66
25	19.76	21.47	23.36	24.72	19.73	20.26	20.93	22.31
26	24.56	28.13	28.25	29.19	23.33	26.31	25.66	28.15
27	22.55	25.9	26.6	25.53	21.32	23.61	24.42	24.3
28	21.76	22.13	23.57	23.62	20.02	20.5	22.54	22.73
29	20.15	22.35	22.09	22.36	20.77	20.88	22.05	22.22
30	20.98	20.87	22.26	22.27	20.33	20.28	21.09	21.88
Avg	21.73	23.25	24.29	25.00	21.18	22.09	23.10	23.82
s.e.	0.36	0.62	0.70	0.77	0.30	0.48	0.60	0.67
s.e.%	1.67%	2.66%	2.87%	3.10%	1.42%	2.19%	2.61%	2.82%

TABLE E-2. Thirty Option Values ($M), P = 300 Sample Paths

	CONIDO Value				Fixed Design Deferral Value			
#	**Π = 3**	**Π = 4**	**Π = 5**	**Π = 6**	**Π = 3**	**Π = 4**	**Π = 5**	**Π = 6**
1	20.82	21.74	21.94	22.31	19.85	20.60	20.71	20.85
2	20.53	21.00	21.08	21.97	19.73	20.28	20.22	20.77
3	21.35	22.74	23.17	22.84	20.31	21.49	21.77	21.83
4	21.23	22.30	23.87	25.76	20.26	21.04	22.56	23.93
5	20.63	21.05	21.17	21.53	19.77	19.73	20.08	20.41
6	21.21	22.04	22.60	23.05	20.09	20.90	21.06	21.71
7	21.03	21.22	21.62	21.78	19.73	20.30	20.72	20.82
8	21.28	21.32	21.47	21.50	20.04	20.60	20.61	20.78
9	20.58	20.96	21.53	21.74	19.73	20.05	20.66	20.98
10	21.35	21.89	21.90	22.21	20.23	20.99	20.89	21.3
11	21.04	22.09	24.78	24.58	20.32	21.91	23.54	23.64
12	21.08	21.75	22.00	22.34	20.15	20.73	21.12	21.22
13	21.06	21.90	21.98	21.96	20.14	20.75	20.86	20.9
14	21.06	21.46	21.62	21.79	20.07	20.48	20.70	21.11
15	21.39	22.04	22.04	21.94	20.47	21.00	20.99	21.04
16	21.57	22.14	22.44	22.38	20.54	21.02	21.00	21.05
17	20.74	21.04	21.49	21.47	19.90	20.15	20.57	20.35
18	22.7	25.48	26.75	28.80	21.44	22.98	24.13	24.29
19	20.56	20.67	21.18	21.20	19.98	20.02	20.42	20.34
20	19.73	19.77	19.96	20.26	19.73	19.73	19.77	19.73
21	20.2	20.56	21.38	21.83	19.73	19.73	20.40	20.60
22	21.58	21.87	23.09	23.75	20.57	20.78	21.83	22.82
23	19.73	19.82	19.87	19.99	19.73	19.73	19.73	19.73
24	21.24	23.25	23.74	26.03	20.36	22.49	22.23	23.19
25	20.48	20.77	21.10	21.52	19.73	19.73	20.00	20.13
26	20.03	20.52	20.62	20.73	19.92	19.87	20.01	20.28
27	20.44	20.75	21.05	21.50	19.76	19.73	19.85	20.47
28	19.73	20.32	20.16	20.31	19.73	19.73	19.73	19.90
29	23.10	27.71	29.12	30.03	21.75	25.74	26.47	27.18
30	21.03	20.85	21.84	22.21	19.83	20.17	20.36	20.68
Avg	20.95	21.70	22.22	22.64	20.12	20.75	21.10	21.40
s.e.	0.14	0.29	0.35	0.42	0.09	0.23	0.27	0.30
s.e.%	0.65%	1.33%	1.59%	1.86%	0.44%	1.10%	1.27%	1.39%

TABLE E-1 and TABLE E-2 empirically the rate of convergence for the algorithm as the number of simulation paths increases from 30 to 300, with the standard error reducing by half.

TABLE E-3. Option Value ($M) as Function of Volatility and No. of Basis Functions *Π*

	CONIDO Value				Fixed Design Deferral Value			
Volatility	***Π* = 3**	***Π* = 4**	***Π* = 5**	***Π* = 6**	***Π* = 3**	***Π* = 4**	***Π* = 5**	***Π* = 6**
5%	19.73	19.73	19.73	19.73	19.73	19.73	19.73	19.73
25%	19.73	19.73	19.73	19.73	19.73	19.73	19.73	19.73
30%	19.73	19.73	19.73	19.76	19.73	19.73	19.73	19.73
35%	21.45	22.34	22.57	22.68	20.40	21.13	21.28	21.50
40%	23.62	23.77	24.04	24.62	22.34	22.63	22.65	23.32
45%	25.54	26.15	26.63	26.98	23.70	24.31	24.01	24.57

As the volatility increases, the NIDO and deferral option values increase. Note that the results of TABLE E-3 are from a single run of LSM simulation at P = 300, so there's a standard error of 1.4 to 1.8 percent associated with the values as indicated in the variance analysis above.

TABLE E-4. 35% Volatility Option Value ($M) as Function of Time Horizon *T* and Number of Basis Functions *Π*

	CONIDO Value				Fixed Design Deferral Value			
Time Horizon (yrs)	***Π* = 3**	***Π* = 4**	***Π* = 5**	***Π* = 6**	***Π* = 3**	***Π* = 4**	***Π* = 5**	***Π* = 6**
0	19.73	19.73	19.73	19.73	19.73	19.73	19.73	19.73
5	21	22	23	23	20	21	21	21
10	44	77	110	133	37	64	83	83
15	268,751	722,357	710,217	861,452	169,822	388,564	325,992	528,441
20	294,324	757,493	741,339	894,429	187,540	407,078	502,526	578,667

As the time horizon increases, the option value increases exponentially at first, but appears to reach an asymptotic value of $894B by the time it's 20 years. The exponential increase is characteristic of the nonlinear congestion effects in the network. The asymptotic behavior is due to the zero drift and 6 percent discount rate having an effect on the future worth of system travel time savings. Note that at such high levels of congestion in the future, more basis functions are needed to estimate the option value in the LSM algorithm.

TABLE E-5. Summary of OLIDOS Solution for 35% Volatility Option Value ($M)

h	h_1	h_2	h_3	h_4	h_5	z_{h_j}	$\sum_{,} \Phi_{h_j}$ ($M)
1	5	4	3	2	1	1	56.29
2	5	4	3	1	2	1	50.96
3	5	4	2	3	1	0	57.17
4	5	4	2	1	3	0	51.54
5	5	4	1	2	3	0	45.42
6	5	4	1	3	2	0	45.46
7	5	3	4	2	1	0	61.87
8	5	3	4	1	2	0	56.40
9	5	3	2	4	1	1	67.93
10	5	3	2	1	4	4	66.66
11	5	3	1	2	4	3	61.05
12	5	3	1	4	2	0	56.15
13	5	2	3	4	1	1	67.82
14	5	2	3	1	4	4	66.68
15	5	2	4	3	1	0	62.83
16	5	2	4	1	3	1	56.61
17	5	2	1	4	3	3	56.38
18	5	2	1	3	4	3	60.86
19	5	1	3	2	4	2	55.20
20	5	1	3	4	2	2	50.48
21	5	1	2	3	4	2	54.77
22	5	1	2	4	3	2	50.58
23	5	1	4	2	3	2	45.54
24	5	1	4	3	2	2	45.48

25	4	5	3	2	1	0	54.61
26	4	5	3	1	2	0	49.76
27	4	5	2	3	1	0	55.50
28	4	5	2	1	3	0	49.70
29	4	5	1	2	3	0	43.59
30	4	5	1	3	2	0	43.51
31	4	3	5	2	1	0	57.80
32	4	3	5	1	2	0	52.94
33	4	3	2	5	1	0	61.92
34	4	3	2	1	5	0	58.20
35	4	3	1	2	5	0	51.71
36	4	3	1	5	2	0	50.19
37	4	2	3	5	1	0	62.83
38	4	2	3	1	5	0	59.55
39	4	2	5	3	1	0	59.53
40	4	2	5	1	3	0	53.81
41	4	2	1	5	3	0	50.34
42	4	2	1	3	5	0	52.66
43	4	1	3	2	5	0	45.58
44	4	1	3	5	2	0	44.00
45	4	1	2	3	5	0	45.23
46	4	1	2	5	3	0	43.26
47	4	1	5	2	3	0	40.61
48	4	1	5	3	2	0	40.40
49	3	4	5	2	1	0	64.83
50	3	4	5	1	2	1	59.32
51	3	4	2	5	1	0	68.84
52	3	4	2	1	5	0	64.78
53	3	4	1	2	5	0	58.80
54	3	4	1	5	2	0	56.97
55	3	5	4	2	1	0	66.83
56	3	5	4	1	2	0	61.70
57	3	5	2	4	1	0	72.99
58	3	5	2	1	4	4	72.41
59	3	5	1	2	4	3	66.81
60	3	5	1	4	2	3	61.74
61	3	2	5	4	1	0	78.67
62	3	2	5	1	4	4	77.88
63	3	2	4	5	1	0	76.32
64	3	2	4	1	5	0	72.85
65	3	2	1	4	5	3	72.18

66	3	2	1	5	4	4	74.26
67	3	1	5	2	4	3	64.62
68	3	1	5	4	2	0	59.62
69	3	1	2	5	4	2	66.83
70	3	1	2	4	5	3	64.45
71	3	1	4	2	5	2	58.89
72	3	1	4	5	2	0	56.85
73	2	4	3	5	1	0	70.04
74	2	4	3	1	5	0	66.66
75	2	4	5	3	1	0	66.91
76	2	4	5	1	3	0	60.99
77	2	4	1	5	3	0	57.37
78	2	4	1	3	5	0	59.80
79	2	3	4	5	1	0	76.34
80	2	3	4	1	5	0	72.87
81	2	3	5	4	1	0	78.35
82	2	3	5	1	4	4	77.89
83	2	3	1	5	4	0	74.34
84	2	3	1	4	5	3	72.23
85	2	5	3	4	1	0	73.84
86	2	5	3	1	4	4	72.46
87	2	5	4	3	1	0	68.28
88	2	5	4	1	3	0	62.24
89	2	5	1	4	3	3	62.24
90	2	5	1	3	4	3	66.60
91	2	1	3	5	4	2	65.70
92	2	1	3	4	5	0	64.01
93	2	1	5	3	4	3	62.95
94	2	1	5	4	3	2	58.78
95	2	1	4	5	3	0	56.01
96	2	1	4	3	5	2	58.40
97	1	4	3	2	5	1	45.76
98	1	4	3	5	2	1	43.60
99	1	4	2	3	5	1	45.20
100	1	4	2	5	3	1	43.15
101	1	4	5	2	3	0	40.31
102	1	4	5	3	2	1	40.35
103	1	3	4	2	5	1	51.80
104	1	3	4	5	2	1	49.55
105	1	3	2	4	5	1	56.92
106	1	3	2	5	4	1	59.24

107	1	3	5	2	4	3	56.28
108	1	3	5	4	2	1	51.88
109	1	2	3	4	5	1	55.94
110	1	2	3	5	4	1	58.15
111	1	2	4	3	5	0	50.53
112	1	2	4	5	3	1	48.49
113	1	2	5	4	3	1	50.87
114	1	2	5	3	4	1	55.21
115	1	5	3	2	4	2	52.90
116	1	5	3	4	2	1	48.17
117	1	5	2	3	4	2	52.54
118	1	5	2	4	3	2	48.31
119	1	5	4	2	3	2	43.26
120	1	5	4	3	2	2	43.25

APPENDIX F. FIRE WEATHER DATA FOR CALIFORNIA

Table F-1. OD Distances of the 12 Nodes used in Chapter 4 Numerical Test Measured in Google Earth (mi, rounded)

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**
1	0	200	90	290	140	180	370	330	260	90	100	140
2	200	0	240	130	100	90	540	500	50	290	150	300
3	90	240	0	360	210	250	300	260	290	90	100	60
4	290	130	360	0	160	110	650	610	100	390	260	420
5	140	100	210	160	0	50	500	460	140	230	130	270
6	180	90	250	110	50	0	550	510	110	280	170	310
7	370	540	300	650	500	550	0	50	590	280	400	240
8	330	500	260	610	460	510	50	0	550	250	350	200
9	260	50	290	100	140	110	590	550	0	350	200	360
10	90	290	90	390	230	280	280	250	350	0	160	90
11	100	150	100	260	130	170	400	350	200	160	0	160
12	140	300	60	420	270	310	240	200	360	90	160	0

Table F-2. Air Tanker Travel Times across 12 Nodes in Chapter 4 Numerical Test assuming 250 mph average speeds (hr)

	1	2	3	4	5	6	7	8	9	10	11	12
1	0.18	0.80	0.36	1.16	0.56	0.72	1.48	1.32	1.04	0.36	0.40	0.56
2	0.80	0.14	0.96	0.52	0.40	0.36	2.16	2.00	0.20	1.16	0.60	1.20
3	0.36	0.96	0.15	1.44	0.84	1.00	1.20	1.04	1.16	0.36	0.40	0.24
4	1.16	0.52	1.44	0.21	0.64	0.44	2.60	2.44	0.40	1.56	1.04	1.68
5	0.56	0.40	0.84	0.64	0.15	0.20	2.00	1.84	0.56	0.92	0.52	1.08
6	0.72	0.36	1.00	0.44	0.20	0.14	2.20	2.04	0.44	1.12	0.68	1.24
7	1.48	2.16	1.20	2.60	2.00	2.20	0.29	0.20	2.36	1.12	1.60	0.96
8	1.32	2.00	1.04	2.44	1.84	2.04	0.20	0.25	2.20	1.00	1.40	0.80
9	1.04	0.20	1.16	0.40	0.56	0.44	2.36	2.20	0.15	1.40	0.80	1.44
10	0.36	1.16	0.36	1.56	0.92	1.12	1.12	1.00	1.40	0.18	0.64	0.36
11	0.40	0.60	0.40	1.04	0.52	0.68	1.60	1.40	0.80	0.64	0.20	0.64
12	0.56	1.20	0.24	1.68	1.08	1.24	0.96	0.80	1.44	0.36	0.64	0.15

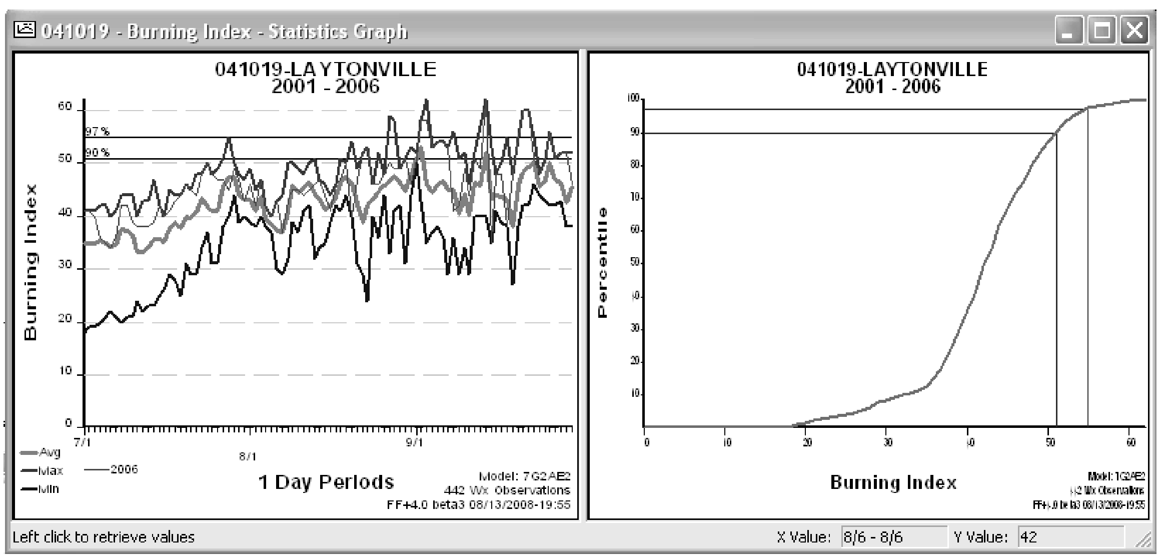

FIGURE F-1. Sample graphical output of data provided by FireFamilyPlus software.

BIBLIOGRAPHY

Abdulaal, M. and L.J. LeBlanc (1979). Continuous equilibrium network design models. *Transportation Research, 13B*, 19-32.

Ahuja, R.K., T.L. Magnanti, and J.B. Orlin (1993). *Network Flows*. Prentice-Hall, Inc., Upper Saddle River, NJ.

Amiro, B.D., K.A. Logan, B.M. Wotton, M.D. Flannigan, J.B. Todd, B.J. Stocks and D.L. Martell (2004), Fire Weather Index System Components for Large Fires in the Canadian Boreal Forest, *International Journal of Wildfire Fire*, 13, 391-400.

Apivatanagul, P. (2008). *Network Design Formulations, Modeling, and Solution Algorithms for Goods Movement Strategic Planning*. PhD Dissertation, University of California, Irvine, 180pp.

Badri, M.A., A.K. Mortagy and C.A. Alsayed (1998), A multi-objective model for locating fire stations. *European Journal of Operational Research*, 110 (2), 243-260.

Bar-Gera, H., Transportation Network Test Problems, http://www.bgu.ac.il/~bargera/tntp/, last update May 17, 2007, accessed June 30, 2009.

Benaroch, M. (2002). Managing information technology investment risk: a real options perspective. *Journal of Management Information Systems, 19 (2),* 43-65.

Benaroch, M. and R.J. Kauffman (2000). Justifying electronic banking network expansion using real options analysis. *MIS Quarterly, 24 (2)*, 197-225.

Berman, O. and A.R. Odoni (1982), Locating Mobile Facilities on a Network with Markovian Properties, *Networks*, 12 (1), 73-86.

Berman, O. and B. LeBlanc (1984), Location-relocation of mobile facilities on a stochastic network. *Transportation Science*, 18 (4), 315-330.

Björkman, M. and K. Holmström (2000). Global optimization of costly nonconvex functions using radial basis functions. *Optimization and Engineering, 1*, 373-397.

Boxall, B. Spending to fight California wildfires tops $1 billion, *Los Angeles Times*, December 31, 2008.

Boyle, P.P. (1977). Options: a Monte Carlo Approach. *Journal of Financial Economics, 4*, 323-338.

Brennan, M. and E. Schwartz (1977). The valuation of American Put Options. *Journal of Finance, 32*, 449-462.

Brotcorne L., G. Laporte and F. Semet (2003), Ambulance location and relocation models. *European Journal of Operational Research*, 147(3), 451-463.

Budiman, M. (2004), Matlab utility: Latin Hypercube Sampling, budiman@acss.usyd.edu.au.

California Department of Forestry and Fire Protection (CDF), http://cdfdata.fire.ca.gov/fire_er/fpp_planning_plans#, Mar 12, 2008.

California Fire Alliance (2008), *Fire Planning and Mapping Tools*, http://wildfire.cr.usgs.gov/fireplanning/, Sept 26, 2008.

Ceylan, H., and M.G.H. Bell (2004). Traffic signal timing optimization based on genetic algorithm approach, including drivers' routing, *Transportation Research Part B*, *38 (4),* 329-342.

Chang, C.H., Y.K. Tung and J.C. Yang (1994), Monte Carlo simulation for correlated variables with marginal distributions, *Journal of Hydraulic Engineering*, 120 (3), 313-331.

Chen, A., H. Yang, H.L. Lo and W.H. Tang (2002). Capacity reliability of a road network: an assessment methodology and numerical results. *Transportation Research Part B, 36 (3)*, 225-252.

Chen, A. and C. Yang (2004). Stochastic transportation network design problem with spatial equity constraint. *Transportation Research Record: Journal of the Transportation Research Board*, *1882*, 97-104.

Chen, A., K. Subprasom and Z. Ji (2006), A simulation-based multi-objective genetic algorithm (SMOGA) procedure for BOT network design problem, *Optimization Engineering*, 7 (3), 225-247.

Chicago Metropolitan Agency for Planning (2007). *Updated 2030 Regional Transportation Plan for Northeastern Illinois*, 260pp.

Chung, Y. (2007). *Development of Spatio-temporal Accident Impact Estimation Model for Freeway Accident Management*. PhD Dissertation, University of California, Irvine, 277pp.

Church, R. and C. ReVelle (1974), Maximal covering location problem, *Papers in Regional Science*, 32 (1), 101-118.

Cox, J.C., A.S. Ross, and M. Rubinstein (1979). Option pricing: a simplified approach. *Journal of Financial Economics, 7*, 229-263.

Cucchiella, F. and M. Gastaldi (2006). Risk management in supply chain: a real option approach. *Journal of Manufacturing Technology, 17 (6)*, 700-720.

Curtis, W., K. Zikan and H. Sowizral (2006), Random Sampling for Multivariate Bernoulli Variables, *United States Patent*, No. US 7,006,954 B1, Feb 28, 2006.

Damnjanovic, I., J. Duthie, and S.T. Waller (2008). Valuation of strategic network flexibility in development of toll road projects. *Construction Management and Economics, 26*, 979-990.

Das, I. and J.E. Dennis (1997), A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems, *Structural and Multidisciplinary Optimization*, 14 (1), 63-69.

Daskin, Mark S (1983), A maximum expected covering location model: formulation, properties and heuristic solution, *Transportation Science*, 17 (1), 48-71.

de Neufville, R. (2000). Dynamic strategic planning for technology policy. *International Journal of Technology Management, 19 (3-5)*, 225-245.

Deb, K., A. Pratap, S. Agarwal, and T. Meyarivan (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *IEEE transactions on evolutionary computation, 6 (2)*, 182-197.

Der Kiureghian, A., and P.L. Liu (1986). Structural reliability under incomplete probability information. *Journal of Engineering Mechanics, 112 (1)*, 85-104.

Dimopoulou, M. and I. Giannikos (2001), Spatial optimization of resources deployment for forest-fire management, *International Transactions in Operational Research*, 8 (5), 523-534.

Dixit, A.K., and R.S. Pindyck (1994). *Investment Under Uncertainty*, Princeton University Press, Princeton, NJ, 468pp.

Drezner, Z. and H. Hamacker (eds) (2002), *Facility Location: Applications and Theory*, Springer.

Du, Z.P. and A. Nicholson (1997), Degradable transportation systems: sensitivity and reliability analysis, *Transportation Research Part B*, 31 (3), 225-237.

Federal Highway Administration (2007). "Citizen's Guide to Transportation Decision Making", http://www.fhwa.dot.gov/planning/citizen/citizen2.htm, Accessed Aug 6, 2009.

Fire and Weather Data, http://fam.nwcg.gov/fam-web/weatherfirecd/california.htm, Sept 27, 2008.

FireFamilyPlus 4.0 beta 3, FireModels.org, USDA, Forest Service, Fire and Aviation Management, Washington, DC, 2000-2008.

Fonseca, C.M. and P.J. Fleming (1993). Genetic algorithms for multiobjective optimization: formulation, discussion and generalization. *Genetic Algorithms: Proceedings of the Fifth International Conference*, San Mateo, CA.

Friesz, T.L., H.J. Cho, N.J. Mehta, R.L. Tobin, and G. Anandalingam (1992). A simulated annealing approach to the network design problem with variational inequality constraints. *Transportation Science*, 26 (1), 18-26.

Frostig, E. (2001), Comparison of portfolios which depend on multivariate Bernoulli random variables with fixed marginals, *Insurance: Mathematics and Economics*, 29 (3), 319-331.

Gamba, A. (2002). An Extension of Least Squares Monte Carlo Simulation for Multi-options Problems. *Proceedings from the 6th Annual International Real Options Conference*, Paphos, Cyprus, July 2002, 41pp.

Gamba, A., and L. Trigeorgis (2007). An improved binomial lattice method for multi-dimensional options. *Applied Mathematical Finance, 14 (5)*, 453-475.

Gao, Z., H. Sun, and H. Zhang (2007). A globally convergent algorithm for transportation continuous network design problem. *Optimization and Engineering*, *8 (3)*, 241-257.

Garvin, M.J. and C.Y.J. Cheah (2004). Valuation techniques for infrastructure investment decisions. *Construction Management and Economics*, *22 (4)*, 373-383.

Goldberg, J.B.(2004), Operations research models for the deployment of emergency services vehicles, *EMS Management Journal*, 1, 20–39.

Greulich, F.E., and W.G. O'Regan (1982). Optimum use of air tankers in initial attack: selection, basing, and transfer rules. *U.S. Department of Agriculture, Forest Service, Research Paper PSW-163*. Pacific Southwest Forest and Range Experiment Station, Berkeley, CA.

Gunasekera, D., G. Mills and M. Williams (2006), Economic impact of fire weather forecasts. *Proceedings RMRS-P-42CD*, US Dept of Agriculture, Forest Service, Rocky Mountain Research Station, 633-637.

Gutmann, H.M. (2001). A radial basis function method for global optimization. *Journal of Global Optimization, 19*, 201-227.

Haight, R.G. and J.S. Fried (2007), Deploying wildland fire suppression resources with a scenario-based standard response model, *INFOR*, 45 (1), 31-39.

Harmantzis, F.C., L. Trigeorgis, and V.P. Tanguturi (2006). Flexible investment decisions in the telecommunications industry: Case applications using real options. *Net Institute, Working Paper #06-06*.

Haykin, S.S. (1994). *Neural Networks: A Comprehensive Foundation*. Maxwell Macmillan International, New York, NY.

Huchzermeier, A. and M.A. Cohen (1996). Valuing operational flexibility under exchange rate risk. *Operations Research, 44 (1)*, 100-113.

Hull, J.C. (2006). *Options, Futures, and Other Derivatives, 6th Ed.* Prentice-Hall, Inc., Upper Saddle River, NJ, 789pp.

Jayakrishnan, R., W.K. Tsai, J.N. Prashker, and S. Rajadhyaksha (1994). A faster path-based algorithm for traffic assignment. *Transportation Research Record: Journal of the Transportation Research Board*, *1443*, 75-83.

Karatzas, I. and S.E. Shreve (1998). *Brownian Motion and Stochastic Calculus, 2nd Ed.* Springer Science + Business Media, LLC, New York, NY, 470pp.

Karoonsoontawong, A. and S.T. Waller (2007). Robust dynamic continuous network design problem, *Transportation Research Record: Journal of the Transportation Research Board, 2029*, 58-71.

Kauffman, R.J. and A. Kumar (2008). Network effects and embedded options: decision-making under uncertainty for network technology investments. *Information Technology and Management, 9 (3)*, 149-168.

Keppo, J. (2002). Optimality with telecommunications network. *IMA Journal of Management Mathematics, 13 (3)*, 211-224.

KNBC Los Angeles. Malibu Fire Claims 35 Homes, Forces 14,000 Evacuations. *KNBC News*, Associated Press, Nov 25, 2007.

Kogut, B. and N. Kulatilaka (1994). Operating flexibility, global manufacturing, and the option value of a multinational network. *Management Science, 40 (1)*, 123-139.

Kolesar, P. and W.E. Walker (1974), An algorithm for the dynamic relocation of fire companies. *Operations Research*, 22(2), 249-274.

Kulatilaka, N. and L. Trigeorgis (2001). The General Flexibility to Switch: Real Options Revisited. *Real Options and Investment under Uncertainty: Classical Readings and Recent Contributions*, ed. E.S. Schwartz and L. Trigeorgis, MIT Press, 179-197.

LeBlanc, L.J. (1975). An algorithm for the discrete network design problem. *Transportation Science*, *9*, 183-199.

Li, H., M.C.J. Bliemer, and P.H.L. Bovy (2007). Optimal toll design from reliability perspective. *Proc., 6th Triennnial Conference on Transportation Analysis*, Phuket, Thailand, 2007.

Lo, H.K. and Y.K. Tung (2003), Network with degradable links: capacity analysis and design, *Transportation Research Part B*, 37 (4), 345-363.

Lockwood, S. (2005). Systems management and operations: A culture shock. *Institute of Transportation Engineers Journal, 75 (5)*, 43-47.

Logi, F. (1999). *CARTESIUS: A Cooperative Approach to Real-Time Decision Support for Multi-Jurisdictional Traffic Congestion Management*. PhD Dissertation, University of California, Irvine, 197pp.

Longstaff, F.A., and E.S. Schwartz (2001). Valuing American options by simulation: a simple least-squares approach. *Review of Financial Studies*, 14 (1), 113-147.

Lou, Y., Y. Yin and S. Lawphongpanich (2009), A robust approach to discrete network designs with demand uncertainty, *Transportation Research Record*, in press.

Luehrman, T.A. (2001). Strategy as a Portfolio of Real Options. *Real Options and Investment under Uncertainty: Classical Readings and Recent Contributions*, ed. Schwartz, E.S. and L. Trigeorgis, The MIT Press, Cambridge, MA, 385-403.

MacLellan, J.I., and D.L. Martell (1996), Basing airtankers for forest fire control in Ontario, *Operations Research*, 44 (5), 677-686.

Magnanti, T.L. and R.T. Wong (1984). Network design and transportation planning: models and algorithms. *Transportation Science, 18 (1)*, 1-55.

Marianov, V. and D. Serra (2002), Location Problems in the Public Sector, in Drezner, Z. and H. Hamacker (eds), *Facility Location: Applications and Theory*, Springer, 119-150.

Marianov, V. and C. Revelle (1991), The Standard response fire protection siting problem, *INFOR*, 29 (2), 116-129.

Markowitz, H. *Mean-Variance Analysis in Portfolio Choice and Capital Markets*. New Hope, PA, 1987.

Marler, R.T. and J.S. Arora (2004), Survey of multi-objective optimization methods for engineering, Structural Multidisciplinary Optimization, 26 (6), 369-395.

Martell, D.L., E.A. Gunn and A. Weintraub (1998), Forest management challenges for operational researchers, *European Journal of Operational Research*, 104 (1), 1-17.

Martell, D.L. (1999) A Markov Chain model of day to day changes in the Canadian forest fire weather index, *International Journal of Wildland Fire,* 9 (4), 265-273.

Mattingly, S.P. (2000). *Decision Theory for Performance Evaluation of New Technologies Incorporating Institutional Issues: Application to Traffic Control Implementation*. PhD Dissertation, University of California, Irvine, 349 pp.

Morlok, E.K. and D.J. Chang (2004). Measuring capacity flexibility of a transportation system. *Transportation Research Part A, 38 (6)*, 405-420.

Mudchanatongsuk, S., F. Ordóñez and J. Liu (2008), Robust solutions for network design under transportation cost and demand uncertainty, *Journal of the Operational Research Society*, 59 (5), 652-662.

Mulvey, J.M., R.J. Vanderbei and S.A. Zenios (1995), Robust optimization of large-scale systems. *Operations Research*, 43 (2), 264-281.

National Fire Danger Rating System (2008), http://www.wrh.noaa.gov/sew/fire/olm/nfdr_ind.htm, Sept 14, 2008

Nelsen, R.B. (2006), *An Introduction to Copulas,* Springer Series in Statistics.
New York Metropolitan Transportation Council (2005). *2005-2030 Regional Transportation Plan*, 94pp.

Ordóñez, F. and J. Zhao (2007), Robust capacity expansion of network flows. *Networks*, 50 (2), 136-145.

Owen, S.H., and M.S. Daskin (1998), Strategic facility location: A review. *European Journal of Operational Research*, 111(3), 423-447.

Pagès, L., R. Jayakrishnan, and C.E. Cortés (2006). Real-time mass passenger transport network optimization problems. *Transportation Research Record: Journal of the Transportation Research Board, 1964,* 229-237.

Patil, G.R., and S.V. Ukkusuri (2007). Modeling flexibility in stochastic transportation network design. *Proceedings from Transportation Research Board 86th Annual Meeting*.

Pichayapan, P., S. Hino, K. Kishi, and K. Satoh (2003). Real option analysis in evaluation of expressway projects under uncertainties. *Journal of the Eastern Asia Society for Transportation Studies*, *5*, 3015-3030.

Potter, B.E., and J.E. Martin (2001), Accuracy of 24- and 48-Hour Forecasts of Haines' Index. *National Weather Digest*, 25 (3-4), 38-46.

Potter, B.E., S. Goodrick and T. Brown (2003), Development of a statistical validation methodology for fire weather indices, *American Meteorological Society*, 11 (1).

Rapp, V. (2004). Western forests, fire risk, and climate change. *Pacific Northwest Research Station*, US Dept of Agriculture, Forest Service.

Regis, R.G., and C.A. Shoemaker (2005). Constrained global optimization of expensive black box functions using radial basis functions. *Journal of Global Optimization*, *31*, 153-171.

Regis, R.G., and C.A. Shoemaker (2007). A stochastic radial basis function method for the global optimization of expensive functions. *INFORMS Journal on Computing*, *19 (4)*, 497-509.

Repede, J.F., and J.J. Bernardo (1994), Developing and validating a decision support system for locating emergency medical vehicles in Louisville, Kentucky, *European Journal of Operational Research*, 75 (3), 567-581.

Sansone, G. (1959). *Orthogonal Functions*, Interscience Publishers, New York.

Saphores, J.D.M., and M.G. Boarnet (2006). Uncertainty and the timing of an urban congestion relief investment. The no-land case. *Journal of Urban Economics*, *59 (2)*, 189-208.

Schütze, O., M. Laumanns, C.A. Coello, M. Dellnitz, and E.G. Talbi (2008). Convergence of stochastic search algorithms to finite size Pareto set approximations. *Journal of Global Optimization, 41 (4)*, 559-577.

Sharma, S., S.V. Ukkusuri and T.V. Mathew (2009), A Pareto optimal multi-objective optimization for the robust transportation network design problem, *Transportation Research Record*, in press.

Sheffi, Y. (1985). *Urban Transportation Networks: Equilibrium Analysis with Mathematical Programming Methods*. Prentice-Hall, Inc., Englewood Cliffs, NJ.

Sheffi, Y. (2001). Supply chain management under the threat of international terrorism. *International Journal of Logistics Management*, *12 (2)*, 1-11.

Shepherd, S. and A. Sumalee (2004), A genetic algorithm based approach to optimal toll level and location problems, *Networks and Spatial Economics*, 4 (2), 161-179.

Shoemaker, C.A., R.G. Regis, and R.C. Fleming (2007). Watershed calibration using multistart local optimization and evolutionary optimization with radial basis function approximation. *Hydrological Sciences Journal, 52 (3),* 450-465.

Shu, J., Teo, C.P., and Z.J.M. Shen (2005). Stochastic transportation-inventory network design problem. *Operations Research, 53 (1)*, 48-60.

Singh, V.P, S. K. Jain and A.K. Tyagi (2007), *Risk and Reliability Analysis: A Handbook for Civil and Environmental Engineers,* ASCE Publications.

Siu, B.W.Y. and H.K. Lo (2008), Doubly uncertain transportation network: Degradable capacity and stochastic demand, *European Journal of Operational Research*, 191 (1), 166-181.

Small, K.A. and E.T. Verhoef (2007), *The Economics of Urban Transportation*, Routledge, London.

Snyder, L.V. (2006). Facility location under uncertainty: a review. *IIE Transactions, 38 (7)*, 547-564.

Snyder, L.V., M.P. Scaparra, M.S. Daskin and R.L. Church (2006). Planning for disruptions in supply chain networks, *Tutorials in Operations Research*, 2006 INFORMS.

Snyder, L.V., M.S. Daskin, and C.P. Teo (2007). The stochastic location model with risk pooling. *European Journal of Operational Research, 179 (3)*, 1221-1238.

Southern California Association of Governments (2008). *Regional Transportation Plan: Transportation Finance Report, Draft 2007*, 54pp.

Spall, J.C. (2003). *Introduction to Stochastic Search and Optimization*. John Wiley and Sons, Hoboken, NJ.

Steenbrink, P.A. (1974), *Optimization of Transport Networks*, John Wiley & Sons.

Stocks, B.J., J.A. Mason, J.B. Todd, E.M. Bosch, B.M. Wotton, B.D. Amiro, M.D. Flannigan, K.G. Hirsch, K.A. Logan, D.L. Martell, W.R. Skinner (2003), Large forest fires in Canada, 1959-1997, *Journal of Geophysical Research, 108 (1)*, 8149-8160.

Sumalee, A., and D.P. Watling (2003). Travel time reliability in a network with dependent link modes and partial driver response. *Journal of Eastern Asia Society for Transportation Studies, 5*, 1687-1701.

Sumalee, A., D.P. Watling, and S.Nakayama (2006). Reliable network design problem: case with uncertain demand and total travel time reliability. *Transportation Research Record: Journal of the Transportation Research Board, 1964*, 81-90.

Sumalee, A., and D.P. Watling (2008). Partition-based approach for estimating travel time reliability with dependent failure probability. *Journal of Advanced Transportation, 42(3)*, 213-238.

Suwansirikul, C., T.L. Friesz, and R.L. Tobin (1987). Equilibrium decomposed optimization: A heuristic for the continuous equilibrium network design problem. *Transportation Science*, *21 (4)*, 254-263.

Tanaka, Katsuto (1996), *Time Series Analysis: Nonstationary and Noninvertible Distribution Theory*, Wiley Series in Probability and Statistics.

Teugels, J.L. (1990), Some representations of the multivariate Bernoulli and Binomial distributions, *Journal of Multivariate Analysis*, 32 (2), 256-268.

Trigeorgis, L. (1991). A log-transformed binomial numerical analysis method for valuing complex multi-option investments. *The Journal of Financial and Quantitative Analysis, 26 (3)*, 309-326.

Trigeorgis, L. (1996). *Real Options: Managerial Flexibility and Strategy in Resource Allocation*. The MIT Press, Cambridge, MA, 427pp.

Tsai, M.T., J.D. Saphores, and A.C. Regan (2008). Freight transportation contracting under uncertainty. UCI-ITS-WP-08-2, ITS Working Paper Series, University of California, Irvine, 15pp.

Tsekeris, T. and S. Voβ (2008), Design and evaluation of road pricing: state-of-the-art and methodological advances, *Netnomics*, Online First.

Ukkusuri, S., T.V. Mathew and S.T. Waller (2007), Robust transportation network design under demand uncertainty, *Computer-Aided Civil and Infrastructure Engineering*, 22 (1), 6-18.

van den Berg, M.A. (2008), Calibrating the Ornstein-Uhlenbeck model. http://sitmo.com/doc/Calibrating_the_Ornstein-Uhlenbeck_model, Sep 18, 2008.

Vergara-Alert, C. (2007). A real option model for optimal investments on transportation. *Proceedings from Transportation Research Board 86th Annual Meeting*.

Verhoef, E.T. (2002), Second-best congestion pricing in general networks. Heuristic algorithms for finding second-best optimal toll levels and toll points, *Transportation Research Part B*, 36 (8), 707-729.

Waller, S.T., and A.K. Ziliaskopoulos (2001). Stochastic dynamic network design problem. *Transportation Research Record*, *1771*, 106-113.

Wei, C.H., and P.M. Schonfeld (1993). An artificial neural network approach for evaluating transportation network improvements. *Journal of Advanced Transportation, 27 (2)*, 129-151.

Welch, W.M. Report: San Diego failing at fire safety. *USA Today*, Feb 19, 2008.

White, H. (1984). *Asymptotic Theory for Econometricians*, Academic Press, New York.

Xiong, Y. and J.B. Schneider (1992). Transportation network design using a cumulative genetic algorithm and neural network. *Transportation Research Record: Journal of the Transportation Research Board, 1364*, 37-44.

Yang, C.H. (2008). *Developing Decision-Making Process for Prioritizing Potential Alternatives of Truck Management Strategies*. PhD Dissertation, University of California, Irvine, 141pp.

Yang, H. and H.K. Lam (1996), Optimal road tolls under conditions of queuing and congestion, *Transportation Research A: Policy and Practice*, 30 (5), 319-332.

Yang, H. and M.G.H. Bell (1997). Traffic restraint, road pricing and network equilibrium. *Transportation Research B, 31 (4)*, 303-314.

Yang, H. and M.G.H. Bell (1998). Models and algorithms for road network design: a review and some new developments. *Transport Reviews, 18 (3)*, 257-278.

Yang, H. and H.J. Huang (1998), Principle of marginal-cost pricing: how does it work in a general road network, *Transportation Research Part A: Policy and Practice*, 32 (1), 45-54.

Yang, H. and Q. Meng (1998), Departure time, route choice and congestion toll in a queuing network with elastic demand, *Transportation Research Part B*, 32 (4), 247-260.

Yang, H. and X. Zhang (2002). Multiclass network toll design problem with social and spatial equity constraints. *Journal of Transportation Engineering, 128 (5)*, 420-428.

Yin, Y. (2008), Robust optimal traffic signal timing, *Transportation Research Part B*, 42 (10), 911-924.

Yin Y., S. M. Madanat and X-Y. Lu (2009), Robust improvement schemes for road networks under demand uncertainty*, European Journal of Operations Research*, 198, 470-479.

Zhao, T., S.K. Sundararajan, and C.L. Tseng (2004). Highway development decision-making under uncertainty: A real options approach. *Journal of Infrastructure Systems*, *10 (1)*, 23-32.

Zitzler, E., L. Thiele, M. LAumanns, C.M. Fonseca, and V.G. da Fonseca (2003). Performance assessment of multiobjective optimizers: an analysis and review. *IEEE Transactions on Evolutionary Computation, 7 (2)*, 117-132.